Special numbers in the United Arab Emirates

A REPORT PREPARED FOR THE TRA UAE

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Executive summary

This report is the final report from the project to advise TRA on the buying and selling of phone numbers. It covers the specific deliverables listed under the terms of reference as set out in Section 1.

The main focus of the project is on ways of categorising and valuing "special" numbers. These are numbers that are so attractive that consumers are willing to pay higher prices for them.

Numbers can be attractive to many people for various reasons; the commonest is that they contain adjacent identical digits (as in '444'), but they might also contain patterns valued in particular cultures (such as '786'), through associations with words (such as 'TAXI') or by particular individuals (such as their dates of birth). A notation for describing these numbers is laid down in Section 2.

The report includes, in Section 3, an extensive benchmarking study of how special numbers are treated in other countries. This establishes that regulators usually manage these numbers in the same ways as other numbers; regulatory activities related to special numbers are very limited and largely concerned with short codes and some other numbers having clear commercial significance.

In particular, no regulator has a process by which consumers can register their mobile numbers to obtain extra rights. However, some regulators ensure that all consumers are protected from losing their numbers inadvertently or unfairly, even if the numbers are not special. Consumer protection of this sort would offer more benefits to more people than special registration and administration processes for special numbers.

Moreover, almost no regulator imposes extra fees for special mobile numbers, though some, perhaps with unfulfilled intentions of imposing fees, have put the numbers into categories ("platinum", "gold", "silver" and so on), with meanings specific to their countries. Where there are extra fees (typically for short codes and other such numbers) they are very different in different countries.

In many countries the service providers, number traders and private individuals participate in vigorous markets in special mobile numbers. Again the prices paid for such numbers vary greatly between (and sometimes within) countries.

Though some regulators have put numbers into categories they have tended to do so in very specific and rather arbitrary ways. These lead to difficulties in determining the quantities and features of those numbers presumed to be attractive. Such difficulties in turn make any potential prices for those numbers, and the relative value to consumers of particular numbers, difficult to assess, because the quantities would affect the prices (through the revenue requirements) and the features would affect the value. General and systematic procedures are needed for categorising, and calculating quantities of, special numbers. This report provides them, based on:

- Formulating patterns of numbers so that formulas that describe them can identify their attractive features, establish the quantities of numbers that fit them, and help to place them in bands according to these quantities (in Section 4).
- Enumerating patterns of numbers so that computations that list and test them can identify their attractive features, establish the quantities of numbers that fit them, and help to place them in bands according to these features (in Section 5).

Even with these general and systematic procedures for categorising special numbers, valuing special numbers remains difficult. Service providers and number traders report prices, not values, and the prices vary greatly: even the largest number traders offer very similar numbers for widely varying prices. Extra fees for special mobile numbers are unlikely to capture all of the factors which influence the values placed by consumers on particular numbers, if they are determined by administrative processes. In theory auctions could reveal these values, but there is only one country where the regulator holds relevant auctions, and even there the auctions are infrequent.

Nonetheless, the relative value of different numbers can be discerned broadly; for instance, a number that repeats a digit six times is likely to command a higher price than a number that repeats the same digit three times. The report follows this up in Section 6, by putting numbers together into price bands. It further indicates how fees could be set for the bands so as to recover number management costs; however, this should be done only in the context of the overall objectives of TRA, such as stimulating competition and husbanding national resources.

The implications of the benchmarking study for numbering policy are discussed further in Section 7.

Findings and recommendations

The following findings and recommendations on numbering policy and the classification and valuation of numbers come from the work described in this report:

1. Number trading is not obviously detrimental (and could even be beneficial) to consumer well-being and should not be subject to regulatory measures, especially as international experience indicates that such measures are difficult to enforce. (Section 7.1.1)

2. The value of special numbers to consumers is difficult to determine without recourse to market mechanisms such as auctions and sales by number traders. Extra fees for special numbers could be created by rebalancing the existing fees to reflect the value broadly while leaving the total revenues to TRA and overall prices to consumers unchanged; revised revenue requirements could be met by scaling the rebalanced fees appropriately. Nonetheless, markets themselves could be more effective than administratively determined fees in ensuring recognition of the value in national resources, and even in raising revenue. (Section 7.1.2)

3. Regulatory auctions for special numbers would not be obviously preferable to the current system in the UAE. Introducing them would require care to avoid giving the impression of competition between the regulator and the service providers and number traders, artificially limiting the supply, disrupting the current secondary market, and raising overall prices paid by consumers. (Section 7.1.3)

4. TRA and perhaps the Consumer Protection Department might need to codify and strengthen the rights of customers, particularly in relation to numbers. (Section 7.2.1)

5. Granting extra rights to customers who buy special numbers could be confusing and complicated. It could also be ineffective: customers would need to renew their rights from time to time (as assigning numbers in perpetuity would cause leaks from the supply) and would forget to do that, just as they forget to renew their subscriptions to the services. Effort could be devoted instead to codifying and strengthening the rights available to all customers, even if they do not buy special numbers. (Section 7.2.2)

6. Number hoarding is not likely to cause problems for the mobile number supply in the UAE and could be controlled by existing regulatory measures. Indeed, it has not been noted as occurring in the UAE. (Section 7.3.1)
7. Allocating punctured blocks is technically feasible but is not especially desirable. Allocating whole blocks, and using the existing processes and systems of the service providers for assigning individual mobile numbers, can achieve the same result. (Section 7.3.2)
8. Examining prices of comparable numbers in different benchmark countries is not likely to be useful for valuing numbers in the UAE. Comparable numbers are on sale in too few countries, and the prices of those that are on sale vary widely. (Section 6.1.1)
9. Relations between the prices of numbers and the attractiveness of numbers are no more precisely defined than the attractiveness itself. However, approximate measures of attractiveness might be used to place numbers in bands ranked broadly according to the value of numbers to customers. The bands could be determined by pattern features such as repetitions, clusters and sequences but not by pattern capacities. (Section 6.1.2)
10. The current annual fees for numbers could be rebalanced to charge more for numbers in higher bands and less for numbers in lower bands. Changing the annual fees in this way would be preferable to introducing one-off fees for special numbers, because it minimises disruption, is fair to everyone who has been or will be assigned numbers, and matches the requirement for rights of use to be renewed from time to time. (Section 6.2.1)
11. The bands determined by pattern features, and the prices appropriate to rebalancing the numbering fees, are robust enough to provide estimates of capacities and revenues that are broadly independent of the choice between the complete eight-digit numbering space and the restricted eight-digit numbering space in which '0' is never the second digit of a number. (Section 6.2.2)
12. The bands formed for the UAE are broadly applicable elsewhere but might match better those used in specific countries if the assumptions about favoured digits, repetitions, clusters and sequences were changed without changing the rules used for forming the bands. (Section 6.3.1)

13. Any proposed relationship between the fees for numbers in different bands should be considered in the light of views of customer demand as expressed by number traders and in consumer surveys and experiments. Moreover, discussions with the number traders would elucidate the effects on consumer behaviour and trading operations of any proposals to change number management for special numbers. (Section 6.3.2)

14. There could be assumptions about favoured digits, repetitions, clusters and sequences that are more appropriate to the UAE than those adopted in much of this report. If they were adopted, the numbers would need to be classified again, at least to the extent needed to put numbers into bands designed according to these more appropriate assumptions. However, the method of classification and the essentials of the final simple rules for forming bands would not be changed. (Section 6.3.3)

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1 Introduction

1.1 Scope

Frontier Economics and Antelope Consulting were commissioned by the Telecommunications Regulatory Authority (TRA) of the United Arab Emirates (UAE) to advise on the selling of numbers. This report, by Robert Milne of Antelope Consulting, is intended to provide the required advice, taking into account discussions with the service providers (Etisalat and Du) as well as with TRA.

The request for proposals identified the following themes in the work:

- Classification methods and formulas for numbers.
- Benchmarking for practices dealing with numbering fees, classification and allocation in a fair manner.
- Valuation of numbers in special classes relative to numbers in normal classes.

The request for proposals also identified six reports (numbered 1, 2, 4, 5, 6 and 7). The themes and these reports are related to the sections of this report as follows:

- Section 2 covers the ground of "report 1" by showing detailed patterns that can be used to categorise numbers.
- Section 3 covers the ground of "reports 2, 4, 5, 6, and 7" by examining, for the benchmark countries, the classification and valuation of numbers (particularly for mobile numbers).
- Section 4 provides classification rules and formulas for numbers that allow the analysis of complex classes.
- Section 5 provides classification rules and enumerations for numbers that can be adopted for simple valuations.
- Section 6 analyses the valuation of numbers in special classes relative to numbers in normal classes.
- Section 7 discusses future national practices in managing numbers, in the light of the benchmarking results on the classification, valuation, allocation and trading of numbers.

The bulk of the examples from the benchmark countries concern mobile numbers, but in some countries the treatments of toll free numbers and short codes have been significant and have evidently influenced the treatments of mobile numbers.

1.2 Structure

The sections of this report are interrelated but are designed to be read separately to some extent. In outline their contents are as follows:

- Section 2 shows detailed patterns that can be used to represent numbers having given lengths. In doing so, it provides rules for calculating the 'capacities' of these patterns, which are the quantities of numbers that the patterns represent; the proportions of the numbering space that the patterns represent can be derived directly from these capacities. The tables of patterns for the benchmark countries in Section 3 should be understood in the light of the conventions and cautions mentioned in this section.

- Section 3 examines, for the benchmark countries, the policies on the classification, valuation, allocation and trading of numbers. It distinguishes between numbers having different resource types (such as toll free, fixed and mobile), as well as between numbers in different categories. It applies the rules developed in Section 2 to establish for the benchmark countries the capacities of the patterns defined by the regulators (and some service providers). It also identifies the fees and rights of use associated with numbers in these categories and examines the patterns recognised and prices charged in the retail market (by service providers, number traders or individual users). It can be understood without understanding all of Section 2.

- Section 4 presents the theory that underpins the investigations in Section 5 and Section 6. It applies that theory in order to classify and band patterns in the complete eight-digit numbering space, by calculating their capacities as well as the capacities of smaller numbering spaces having numbers with four, five, six or seven digits. In doing so it demonstrates a distinctive new basis for the analysis of special numbers. A slight familiarity with mathematical ideas would help with understanding it.

- Section 5 presents techniques that are built on part of the theory in Section 4 but that can be used instead of extensions to that theory. It applies the techniques to calculate the capacities of further patterns and develop different ways of classifying and banding numbers. Among the examples treated fully are both the complete eight-digit numbering space and a restricted eight-digit numbering space in which '0' is never the second digit of a number. The techniques can be applied without a full appreciation of the theory, or even the notation, of Section 4.

- Section 6 uses the results from the benchmark countries provided in Section 3, the theory established in Section 4 and the techniques introduced in Section 5 to investigate ways of pricing numbers and the applicability of these ways in the UAE (particularly for mobile numbers). It shows how the patterns representing special numbers that could be appropriate to the UAE can be divided among potential price bands. It can be understood without understanding much of Section 4.
- Section 7 provides suggestions about the policy implications for the UAE of special number management. It reaches conclusions about buying and selling numbers, protecting consumers, and safeguarding the number supply that draw on the findings in the other sections (particularly those about the benchmark countries in Section 3). However, it can be read without reading those sections.

1.3 The nature of special numbers

Special numbers are phone numbers that users like for any of the following reasons:

- They can be read as words because of the standard mapping of digits to letters on key pads, as can '8294' ("TAXI").
- They can be read as words because of their appearance, as can '8055' ("BOSS").
- They can be pronounced as words by saying the digits separately, as can '2' ("TO"), '4' ("FOR") and '8' ("ATE").
- They can be pronounced as words by saying the digits together, as can '20' ("TWENTY"), '40' ("FORTY") and '80' ("EIGHTY").
- They have some particular cultural significance, as have '88' ("doubly fortunate"), '138' ("rich during your life") and '168' ("wealthy for all time") in China, '786' ("bismallah") in Pakistan, '147' (the maximum snooker break) and '180' (the maximum darts score) in the UK and '911' (the emergency code) and '247365' (shop opening hours) in the United States (US).
- They have some particular personal significance (such as identifying a date of birth or marriage).
- They have attractive patterns of digits.

This report concentrates on the last reason for being special, that the numbers have attractive patterns of digits. Such numbers are in practice the most popular special mobile numbers (though possibly not the most popular special toll free numbers, which can sometimes be seen or pronounced as words).

1.4 The choice of benchmark countries

For each of sixteen benchmark countries (and the UAE) the report summarises the relevant features of the numbering plan, numbering fees and number allocation rules (including particular consumer protection aspects) and tabulates the main categories of special numbers as defined by the regulator, service providers and number traders.

The benchmark countries are drawn from the following groups:

- Members of the Organisation for Economic Development and Co-operation (OECD): Australia, Belgium, France, Ireland, Norway and the United Kingdom (UK).
- Members of the Gulf Co-operation Council (GCC): Bahrain, Kuwait, Oman, Qatar and Saudi Arabia, as well as the UAE.
- Countries having service providers that are subsidiaries of those in the UAE: Egypt, Pakistan and Saudi Arabia.
- Countries doing something slightly unusual about special numbers: Hong Kong and Singapore.
- Countries having many citizens in the UAE: Egypt, Lebanon and Pakistan.

In all of these countries end users sell mobile numbers, through classified advertisements on web sites and elsewhere. In several of them service providers and number traders sell mobile numbers (though, formally, packages, not numbers, might be the items sold). In relatively few countries regulators have categories of special mobile numbers, and in even fewer countries regulators allocate special mobile numbers for higher prices (whether fixed in advance or determined by auction).

1.5 The national numbering plan

There is a restriction on the numbering space in the UAE related to the use of numbers with '0' in the second position of the eight digits following '05': in the past TRA has allocated such numbers free of charge to service providers only for their own internal purposes. There appear to be few reasons why this restriction on use is needed now, and if numbers were for sale the restriction would probably be regarded as an artificial impediment to supply. Its effect on the quantities of special numbers therefore deserves attention.

Both the 'complete' eight-digit numbering space, comprising all of the numbers having eight digits following '05', and a 'restricted' numbering space, comprising all of those numbers without '0' in the second position, are considered in this report.

2 Describing numbers

This section shows detailed patterns that can be used to represent numbers having given lengths. In doing it provides rules for calculating the 'capacities' of these patterns, which are the quantities of numbers that the patterns represent; the proportions of the numbering space that the patterns represent can be derived directly from these capacities. The tables of patterns for the benchmark countries in Section 3 should be understood in the light of the conventions and cautions mentioned in this section.

2.1 Notation

2.1.1 Patterns in numbers

Numbers are made attractive by having:

- Few distinct digits (as in '24422442', '24742724' and '24752725', where there are respectively two, three and four distinct digits).
- Particular favoured digits (as in '20704090', '02704059' and '27040859', where there are respectively four, three and two occurrences of '0', which is assumed to be favoured).
- 'Repetitions' that comprise adjacent digits which are identical with each other (as in '22222247', '27222259' and '24742259', where there are respectively six, four and two occurrences of '2' adjacent to each other).
- 'Clusters' that comprise adjacent digits which also occur in a 'similar' arrangement as the digits of another cluster in the same number (as in '24752475', '24782479' and '24782459', where the other clusters with similar arrangements to the clusters '2475', '247' and '24' in the three numbers are in fact identical with those clusters).
- 'Sequences' that comprise adjacent digits which are related by arithmetic rules (as in '01234567', '76543210', '81234569', '86543219', '12383219' and '65484569', where the sequences '01234567', '76543210', '123456', '654321', '123' and '654' are in fact arithmetic progressions ascending or descending in steps of one).

The examples of repetitions, clusters and sequences are only indicative; different examples might be appropriate to different countries. For instance:

- Repetitions might be accepted if two digits had two occurrences adjacent to each other (as in '22477859'), even though they would not be if only one digit had two occurrences.

- Clusters might be accepted if they were similar to each other but not identical, in that they reversed or permuted the digits of each other (as in '27454729' and '27452479').
- Sequences might be accepted if the arithmetic rules produced arithmetic progressions ascending or descending in steps of two (as in '02461357' and '75316420') or even repetitions or permutations of the digits (as in '22223345' and '22522343').

Describing numbers so that these attractive features of patterns can be understood and generalised requires developing a notation for them, such as that below.

2.1.2 Representations of numbers

In this report telephone numbers are represented using 'patterns' with optional 'constraints'. Patterns provide representations for the digit in the numbers; constraints restrict the digits allowed.

The digits are represented in a pattern by numerals ('0', '1', '2' and so on), by lower-case letters without subscripts ('a', 'b', 'c' and so on), by lower-case letters with subscripts ('a_0', 'b_3', 'c_7' and so on) or by occurrences of '*'. For instance:

- The pattern '2110' represents a specific four-digit number.
- The pattern 'aaaa' represents four-digit numbers that might be restricted by constraints on 'a'.
- The pattern 'abcd' represents four-digit numbers that might be restricted by constraints on 'a', 'b', 'c' and 'd' (and indeed are already implicitly restricted as mentioned in Section 2.2.1).
- The pattern '$a_0a_1a_2a_3$' represents four-digit numbers that might be restricted by constraints on 'a_0' (and indeed are already implicitly restricted as mentioned in Section 2.2.1) and that form arithmetic progressions ascending in steps of one.
- The pattern '****' represents four-digit numbers that are not restricted by explicit constraints (but that might already have been restricted implicitly as discussed in Section 2.3.2).

These representations of digits can be mixed. For instance:

- The pattern 'ab*0' represents four-digit numbers that might be restricted by explicit constraints on the first and second digits, that are not restricted by explicit constraints on the third digit, and that have '0' as the fourth digit; thus if 'a' represented '2', 'b' represented '1' and '*' was not already restricted then 'ab*0' would represent '2110'.

- The pattern 'a_0b*a_3' represents four-digit numbers that might be restricted by explicit constraints on the first and second digits, that are not restricted by explicit constraints on the third digit, and that have a fourth digit which is three greater than the first digit; thus if 'a_0' represented '2', 'b' represented '1' and '*' was not already restricted then 'a_0b*a_3' would represent '2115'.

In several countries sequences are arithmetic progressions ascending or descending in steps of one; in some countries the steps can be greater than one. The notation in this report is intended to allow these, and other, possibilities. It assumes that digits stop at '9' (though in at least one country '0' is regarded as the next digit beyond '9').

Different letters without subscripts in a pattern represent different digits. For instance:

- The pattern 'ab*0' might represent '2100', if the constraints allow this, but does not represent '2200'.

Different occurrences of the same letter without subscripts in a pattern represent the same digit. For instance:

- The pattern 'aa*0' might represent '2200', if the constraints allow this, but does not represent '2100'.

Different occurrences of the same letter with the same subscripts in a pattern represent the same digit. For instance:

- The pattern 'a_0a_0*0' might represent '2200', if the constraints allow this, but does not represent '2100'.

Different occurrences of the same letter with different subscripts in a pattern represent digits that differ by the same amount as the subscripts. For instance:

- The pattern 'a_0a_1*0' might represent '2300', if the constraints allow this, but does not represent '2100'.

Different occurrences of '*' in a pattern do not necessarily represent the same digit. For instance:

- The pattern 'a**0' might represent '2100' or '2110', if the constraints allow this, but does not represent '2111'.

2.1.3 Constraints on patterns

Letters represent digits, so there are implicit constraints on them; for example, 'a' is implicitly subject to '$a \geq 0$' and '$a \leq 9$'. Where the occurrences of a letter in a pattern have subscripts, all of the occurrences are implicitly constrained, so the occurrence of the letter with the highest subscript in the pattern constrains all of the occurrences of the letter. For instance:

- The pattern 'a_0b*a_3' is implicitly restricted by the constraint '$a_3 \leq 9$' so it is implicitly restricted by the constraint '$a_0 \leq 6$'.

In this report there is also a convention that different letters without subscripts represent different digits. For instance:

- The pattern 'ab*0' is implicitly restricted by the constraint 'b≠a'.

Even where there are implicit constraints there can be explicit constraints, too; these typically relate letters such as 'b' to numerals such as '0', '1' or '2' through formulas such as 'b=0', 'b≠2', 'b≥1' and 'b≤3'. For instance:

- The pattern 'ab*0' with the constraint 'b≠2' might represent '2100', if the other constraints allow this, but does not represent '1200'.

Explicit constraints are written as parts of the patterns, separated from the representations of the digits by 'with'. However, explicit constraints are not very common: many useful patterns have no constraints, and no useful patterns have many constraints.

Constraints can be combined using 'and': the combined constraint holds when all of the constraints in it hold. For instance:

- The pattern 'ab*0' with the constraint 'a≠1 and b≠2' might represent '2100', if the other constraints allow this, but does not represent '1200', '1300' or '3200'.

In addition, constraints can be combined using 'or': the combined constraint holds when at least one of the constraints in it holds. For instance:

- The pattern 'ab*0' with the constraint 'a≠1 or b≠2' might represent '2100', '1300' or '3200', if the other constraints allow this, but does not represent '1200'.

2.2 Capacity calculation

2.2.1 Capacities of patterns

Different numbers might be represented by the same pattern; the 'capacity' is how many numbers the pattern represents. The proportion of the numbering space that a pattern represents can be obtained by dividing the capacity of the pattern by the size of the numbering space. Thus in the numbering space comprising 10,000 four-digit numbers a pattern having capacity 900 represents 900/10,000, or 9%, of the numbers.

The digits allowed in the numbers in a pattern can be restricted by constraints, so the capacity of the pattern can be reduced by imposing constraints. If there are no constraints then all digits (from '0' to '9') are allowed. For instance:

- The pattern '2110' represents one four-digit number.

- The pattern ‘aaaa’ represents 10,000 four-digit numbers if there are no constraints on ‘a’.

- The pattern ‘****’ represents 10,000 four-digit numbers, because each ‘*’ can represent ten different digits if there are no constraints.

The remark “if there are no constraints” in rules for calculating capacities involving ‘*’ can be ignored until the explanation in Section 2.3.2.

Often there are implicit or explicit constraints. For instance:

- The pattern ‘$a_0a_1a_2a_3$’ represents 7 four-digit numbers (which are ‘0123’, ‘1234’, ‘2345’, ‘3456’, ‘4567’, ‘5678’ and ‘6789’) if there are no constraints on ‘a_0’ (other than the implicit constraint ‘$a_3 \leq 9$’ that entails the implicit constraint ‘$a_0 \leq 6$’).

In particular, if there are different letters without subscripts in a pattern then when one of the letters represents a particular digit the other letters are not allowed to represent that digit. For instance:

- The pattern ‘abcd’ represents 10×9×8×7 (or 5,040) four-digit numbers if there are no constraints on ‘a’, ‘b’, ‘c’ or ‘d’ (other than the implicit constraint ‘b≠a and c≠b and c≠a and d≠c and d≠b and d≠a’).

The capacity of a pattern can be calculated by multiplying the capacities of its numerals, letters and occurrences of ‘*’. Each numeral has capacity one; the letters and occurrences of ‘*’ have collectively a capacity that takes into account any constraints on them. For instance:

- The pattern ‘ab*0’ represents the four-digit numbers formed by restricting the digits represented by ‘ab’ according to the constraints, letting ‘*’ represent all digits from ‘0’ to ‘9’ if there are no additional constraints, and making the fourth digit be ‘0’, so its capacity is ten times the capacity of ‘ab’.

- The pattern ‘$a_0b{*}a_3$’ represents the four-digit numbers formed by restricting the digits represented by ‘a_0b’ according to the constraints, letting ‘*’ represent all digits from ‘0’ to ‘9’ if there are no additional constraints, and making the fourth digit three greater than the first digit, so its capacity is ten times the capacity of ‘a_0b’.

Different letters without subscripts in a pattern represent different digits. For instance:

- The pattern ‘ab*0’ represents 900 four-digit numbers if there are no constraints (other than the constraint ‘b≠a’): ‘a’, ‘b’ and ‘*’ each represent all digits from ‘0’ to ‘9’ except that ‘a’ and ‘b’ must represent different digits, so there are ten digits represented by ‘a’ but for each of these there are only nine digits represented by ‘b’.

Different occurrences of the same letter without subscripts in a pattern represent the same digit. Consequently:

- The pattern 'aa*0' represents 100 four-digit numbers if there are no constraints: 'a' and '*' each represent all digits from '0' to '9'.

Different occurrences of the same letter with the same subscripts in a pattern represent the same digit. Consequently:

- The pattern 'a_0a_0*0' represents 100 four-digit numbers if there are no constraints: 'a_0' and '*' each represent all digits from '0' to '9'.

Different occurrences of the same letter with different subscripts in a pattern represent digits that differ by the same amount as the subscripts. Consequently:

- The pattern 'a_0a_1*0' represents 90 four-digit numbers if there are no constraints (other than the constraint '$a_1 \leq 9$' which entails the constraint '$a_0 \leq 8$'): 'a_0' represents all digits from '0' to '8' and '*' represents all digits from '0' to '9'.

Different occurrences of '*' in a pattern do not necessarily represent the same digit. Consequently:

- The pattern 'a**0' represents 1,000 four-digit numbers if there are no constraints: 'a' and '*' each represent all digits from '0' to '9'.

Imposing constraints on a pattern reduces the capacity of the pattern. For instance:

- The pattern 'ab*0' with the constraint '$a \neq 1$ and $b \neq 2$' represents 730 four-digit numbers if there are no other constraints if there are no other constraints (other than the constraint '$b \neq a$'): 'a' and 'b' represent all digits from '0' to '9' except that 'a' and 'b' must represent different digits and 'a' must not represent '1' and 'b' must not represent '2', so there are nine digits represented by 'a' and eight digits represented by 'b' (unless 'a' represents '2', when there are nine digits represented by 'b') or nine digits represented by 'b' and eight digits represented by 'a' (unless 'b' represents '1', when there are nine digits represented by 'a').

In addition, constraints combined using 'or' are treated similarly. For instance:

- The pattern 'ab*0' with the constraint '$a \neq 1$ or $b \neq 2$' represents 890 four-digit numbers if there are no other constraints (other than the constraint '$b \neq a$'): 'a' and 'b' represent all digits from '0' to '9' except that 'a' and 'b' must represent different digits and 'a' must not represent '1' or 'b' must not represent '2', so there are ten digits represented by 'a' and nine digits represented by 'b' (unless 'a' represents '1', when there are eight digits represented by 'b') or ten digits represented by 'b' and nine digits represented by 'a' (unless 'b' represents '2', when there are eight digits represented by 'a').

2.2.2 Rules for capacities

In calculating the capacity of a pattern by multiplying the capacities of its numerals, letters and occurrences of '*', simple rules are:

- The capacity of a numeral is one.
- The capacity of an occurrence of '*' is ten if there are no constraints.
- The capacity of all of the occurrences of a letter except the first is one.
- The capacity of the first occurrence of a letter without a subscript is ten if there are no constraints.
- The capacity of the first occurrence of a letter with a subscript is the difference between ten and the highest subscript of the letter in the pattern if there are no constraints.

Constraints on patterns can usually be formed by combining constraints typified by 'b=0', 'b≠2', 'b≥1' and 'b≤3'. In fact:

- The capacity of the first occurrence of 'b' is one if the only constraint on 'b' is 'b=0' (as 'b' can represent '0', which would often be written instead of 'b' in the pattern).
- The capacity of the first occurrence of 'b' is nine if the only constraint on 'b' is 'b≠1' (as 'b' can represent '0', '1', '3', '4', '5', '6', '7', '8' or '9').
- The capacity of the first occurrence of 'b' is nine if the only constraint on 'b' is 'b≥1' (as 'b' can represent '1', '2', '3', '4', '5', '6', '7', '8' or '9').
- The capacity of the first occurrence of 'b' is four if the only constraint on 'b' is 'b≤3' (as 'b' can represent '0', '1, '2' or '3').

In general combining constraints involves using 'and' and 'or'. In practice the constraints on a pattern can be written as combinations (using 'or') of other constraints only one of which holds in any circumstance; moreover, each of these other constraints is itself a combination (using 'and') of constraints typified by 'b=0', 'b≠2', 'b≥1' and 'b≤3'.

When a pattern is constrained by a combination (using 'or') of separate constraints only one of which holds in any circumstance, the capacity of the pattern constrained by the combination can be calculated by adding the capacities of the pattern constrained by the separate constraints.

When a pattern is constrained by a combination (using 'and') of separate constraints which have no letters in common, the capacity of the pattern constrained by the combination can be calculated by multiplying the capacities of patterns obtained by splitting the pattern into separate patterns, each of which is constrained by only one of the separate constraints and the convention that different letters represent different digits.

2.3 Usage in this report

2.3.1 Terminology

In this report there are the following sorts of sets of numbers:

- 'Categories' are the sets described, defined or enumerated already by regulators, service providers or number traders. They might have names such as 'gold', 'silver' and 'bronze' and might associate prices with the numbers in them. Usually there are few of them in the numbering space.
- 'Classes' are the sets defined in this report to establish the quantities of numbers having particular combinations of attractive features. Because the features can be combined in many ways there can be hundreds of classes in the numbering space.
- 'Bands' are the sets defined in this report to merge and sort the classes into entities that can be ranked broadly according to the value of numbers to customers. Because they are ranked in imitation of the presumed overall feeling of customers there are few of them in the numbering space.

2.3.2 Conventions in tables of patterns

The 10,000 four-digit numbers can be represented, in possibly decreasing order of attractiveness, by the following patterns:

- The pattern 'aaaa' has capacity 10 (and represents 0.1% of the numbers).
- The patterns 'aaab', 'aaba', 'abaa' and 'baaa' (and each represent 0.9% of the numbers).
- The patterns 'aabb', 'abab' and 'abba' each have capacity 90 (and each represent 0.9% of the numbers).
- The patterns 'aabc', 'abac', 'abca', 'baac', 'baca' and 'bcaa' each have capacity 720 (and each represent 7.2% of the numbers).
- The pattern 'abcd' has capacity 5040 (and represents 50.4% of the numbers).

The four-digit numbers in the example above fall into five groups. Table 1 displays the patterns and capacities accordingly.

Table 1 The four-digit patterns represented by letters only

Pattern	Capacity	Proportion (%)
aaaa	10	0.1
aaab	90	0.9
aaba	90	0.9
abaa	90	0.9
baaa	90	0.9
aabb	90	0.9
abab	90	0.9
abba	90	0.9
aabc	720	7.2
abac	720	7.2
abca	720	7.2
baac	720	7.2
baca	720	7.2
bcaa	720	7.2
abcd	5,040	50.4

Table 1 adopts the convention, followed in this report, that different letters without subscripts in a pattern necessarily represent different digits. This convention entails distinguishing between patterns in which two letters must represent different digits and patterns in which they must not do so. Hence it sometimes requires the insertion of more patterns into tables and the enumeration of more cases in the calculation of capacities; some of the tables for the benchmark countries are made quite long under this convention. However, it simplifies reasoning in the situations of most interest in this report.

Some of the tables for the benchmark countries also need to accommodate patterns with complicated constraints, typically for short codes and some other numbers having clear commercial significance. For these specific cases there are more compact notations than the ones apparent in this report, which is intended to provide a general and systematic treatment of special numbers.

Table 1 is designed so that the patterns in each row do not represent any of the numbers represented by patterns in earlier rows. The letters ‘a’, ‘b’, ‘c’ and ‘d’ can therefore sometimes be regarded as unnecessary, as the constraints on them (especially the implicit constraints that distinct letters represent distinct digits) are implied by the ordering of the rows. Table 2 displays the patterns and capacities leaving out letters than can be regarded as unnecessary.

Table 2 The four-digit patterns represented by letters and '*'

Pattern	Capacity	Proportion (%)
aaaa	10	0.1
aaa*	90	0.9
aa*a	90	0.9
a*aa	90	0.9
*aaa	90	0.9
aabb	90	0.9
abab	90	0.9
abba	90	0.9
aa**	720	7.2
a*a*	720	7.2
a**a	720	7.2
aa	720	7.2
*a*a	720	7.2
**aa	720	7.2
****	5,040	50.4

In tables following the style of Table 2, patterns use letters only to indicate that digits are identical or are restricted with explicit constraints such as 'b=0', 'b≥1' and 'b≤3'. Other letters are replaced by '*' to avoid distracting attention from the important properties of the pattern.

However, tables following the style of Table 2 leave so much to be inferred from the ordering of the rows that they can make the patterns difficult to interpret and the capacities difficult to calculate: the meaning of a pattern depends on the meanings of the patterns in earlier rows, and there can be many such rows. In fact, in this style even '*' does not have a simple meaning. For instance, the four occurrences of '*' in the final row of Table 2 necessarily represent different digits, but in other tables they might not do so. That is why the remark "if there are no constraints" in rules for calculating capacities applies to '*' as well as to letters.

Though '*' is used in the tables for some of the benchmark countries, it is not used in the detailed analyses of the eight-digit numbering space that depend on precise definitions of patterns outside the context of tables.

2.3.3 Cautions about benchmarks

This report includes descriptions of the numbering practices in benchmark countries. However, these need to be treated with caution, as they are intended only to summarise the main features of the practices relevant to the current study. In particular, the outlines of the national numbering plans in the benchmark countries are approximate: they indicate the main uses of some, but not all, of the leading digits and they ignore most exceptions to those uses.

Other limitations of the descriptions of the numbering practices are as follows:

- The tables of special numbers that contain capacities are derived from exhaustive descriptions, definitions or enumerations of the special numbers. However, the patterns and constraints might not be what the originators intended, as for some of the original descriptions there appear to be:
  - Omissions (such as excluding 'aabbbb**' despite including '**aabbbb', '*aa*bbbb', 'aa**bbbb', '*aabbbb*' and 'aa*bbbb*').
  - Ambiguities (such as being unclear about which of '*aaa*bbb' and 'ab*ab*ab' to include).
  - Duplications (such as putting '**ababab' in two categories).
  - Oddities (such as putting 'abababab' and 'ababab**' in the same category).
- The capacities calculated for the patterns in the tables might not be what the originators expected, either because the patterns and constraints themselves are not what the originators intended or because the capacities have been calculated without taking into account the fact that some parts of the national numbering space (such as numbers beginning with '00', in many countries) are never to be allocated. Moreover, the capacities are calculated after omitting the national prefix '0' and any call-by-call selection code, where those exist.
- The tables of patterns that contain prices are derived by sampling the web sites of the sellers. They give many examples of prices and patterns but are not exhaustive. The patterns suggested in this report might not be the same as those noticed by the sellers when attaching prices to the numbers, and the prices might be anomalous. For instance:
  - The sellers might consider '000' to be much more attractive than '222', in which case representing '000' as the pattern 'aaa' in the tables would be inappropriate; sequences are particularly problematic in this regard, as, for example, the patterns '$a_0a_2a_1$', '$a_1a_2a_0$' and '$a_8a_9a_0$' might be inappropriate representations of '132', '786' and '890' respectively.
  - The sellers have sometimes deliberately or inadvertently attached different prices to numbers that are different but that can be represented by the same pattern.
  - The sellers have sometimes devised prices to accord with the numbers themselves in some way; among the examples are AED 777,770 for '567777770', USD 682 for '70001682' and GBP 7,867.86 (now reduced to GBP 3,000) for '786*786786'.

  - The sellers can change the prices easily to reflect supply and demand; some prices appear to have been changed (or at least removed from the web sites) even by the observations made for this report.

3 Number management practices

This section examines, for the benchmark countries, the policies on the classification, valuation, allocation and trading of numbers. It distinguishes between numbers having different resource types (such as toll free, fixed and mobile), as well as between numbers in different categories. It applies the rules developed in Section 2 to establish for the benchmark countries the capacities of the patterns defined by the regulators (and some service providers). It also identifies the fees and rights of use associated with numbers in these categories and examines the patterns recognised and prices charged in the retail market (by service providers, number traders or individual users). It can be understood without understanding all of Section 2.

3.1 United Arab Emirates

3.1.1 Background

The regulator is the Telecommunications Regulatory Authority (TRA). The main service providers are Etisalat and Du.

In national dialling, fixed and mobile numbers are prefixed by '0'. The numbering plan of 2008/1429 provides:

- Fixed numbers beginning with '2', '3', '4', '6', '7' or '9' and having eight digits.
- Mobile numbers beginning with '5' and having nine digits.
- Toll free numbers beginning with '800' and having five, six, seven, eight, nine, ten, eleven or twelve digits.
- Premium rate numbers beginning with '900' and having nine digits.
- Short codes for general services beginning with '1' and having three digits.
- Short codes for emergency services beginning with '99' and having three digits.
- Short codes for text messages beginning with '0', '1', '2', '3', '4', '5' '6', '7', '8' or '9' and having four digits.

There are also free access, shared cost, shared revenue, emergency, paging and international toll free numbers.

3.1.2 Fixed and mobile numbers

TRA allocates fixed numbers in blocks of 1,000 and mobile numbers in blocks of 10,000 or 100,000. There are an application fee of AED 500 for each application, and an annual fee of AED 0.4 for each fixed number or AED 2.0 for each mobile number. If a specific fixed number is requested the annual fee is AED 1.2 instead of AED 0.4. There are no categories of special numbers.

Mobile numbers may be ported.

Etisalat includes special numbers in particular tariff packages. For these packages Etisalat lets customers select special ('VIP') mobile numbers represented by **aaaa**a (called 'platinum') or **aaaa*** (called 'gold'). Estimating the prices of these numbers from the prices for the packages is not straightforward. However, the difference between the first and second year prices for a package can be taken to be the price of the special number and the handset. Moreover the handset can be one priced at up to AED 3,500 for a platinum number and one priced at up to AED 1,000 for a gold number (according to Etisalat customer services). On this basis a platinum number is priced at AED 9,000 and a gold number is priced at AED 3,250. However, after two years the package needs to be renewed, presumably for the original first year price, so the annual price for use during one year is in effect AED 4,500 for a platinum number and AED 1,625 for a gold number.

Etisalat also sells special mobile numbers in auctions for charity; Table 3 exhibits the prices at one such auction in 2014/1435.

Table 3 Special mobile numbers as priced in an auction in the United Arab Emirates

Price	Number
AED 7,877,777	507777777
AED 1,250,000	507777770
AED 900,000	507777771
AED 900,000	507777776
AED 900,000	507777778
AED 800,000	507777775
AED 620,000	507770777
AED 570,000	507777788
AED 18,000	507770011

Du includes special numbers in particular tariff packages. For these packages Du lets customers select special ('executive') mobile numbers represented by **aaaa*** (called 'gold'). Estimating the prices of these numbers from the prices for the packages is not feasible; Du claims that its gold numbers are worth an annual payment of AED 9,000 each; however, Etisalat platinum numbers are implicitly priced at AED 4,500, according to the estimate above.

Du lets customers select special mobile numbers from lists with minimum prices of AED 100; however, the customers must make monthly payments for telecommunications services of at least AED 750 (to be offered gold numbers), AED 500 (to be offered silver numbers), AED 250 (to be offered bronze numbers) or AED 100 (to be offered other numbers). Du also sells special mobile numbers in auctions for charity.

Companies such as 050 VIP Numbers, UAE VIP Numbers and Autotrader UAE let customers select special mobile numbers from lists. The lists from Autotrader UAE, sampled for Table 4, are easily the most extensive.

Table 4 Special mobile numbers as defined by one number trader in the United Arab Emirates

Category	Price	Pattern
	AED 777,770	**aaaaaa0
	AED 580,000	*0*00000*
	AED 200,001 – AED 300,000	**aaaaaa* **aaaa*aa **aa*aaaa **a**aaaa ****aaaaa *0*0aaaaa
	AED 100,001 – AED 200,000	aaaaaa*a* aaaaaaa** aaaa*0000 **aaaaa** ***aaaaaa **a0a0a0a *0*0*000* **aaaaabb aaaaaabab aaaaabbba aa0bba000 aab0b0000
	AED 50,001 – AED 100,000	aaaaaaa*a* aaaa0aa0* aaaaa*000 aabb0b0bb aab*bb*bb aabbbbb** aa*bbbbb* aa**bbbbb
	AED 20,001 – AED 50,000	aa*000*00 aa*00*000 **a000a00 aaabbbbb* aabaaabbb

		aa*aabbbb aabb*0000 aa*bb0000 aa***bbbb **aaabbbb aabbccbbb aabccbbbb aabccbcbb aa*bcccbb
	AED 5,001 – AED 20,000	$a_0a_0a_0a_1a_2a_2a_2a_3a_3$ **aaaa*a* aaa0a0a0* aaaaab*b* aaa**bbbb aabbbb**b aab**bbbb aa***bbbb *a*bbb*ba aaaabcbcb aabccbbcc aabbbbcc* aabbccc*b aa**bbccc aabcbbcbc* aa***bcbc

There are many classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 5.

Table 5 Special mobile numbers as priced in classified advertisements in the United Arab Emirates

Price	Number
AED 13,000	507487474
AED 10,000	527864876
AED 5,000	501023033
AED 2,500	505888569
AED 2,000	508408485
AED 1,000	502012320
AED 500	567181066
AED 200	553388234

3.1.3 Toll free and premium rate numbers

TRA allocates toll free and premium rate numbers individually. There are an application fee of AED 500 for each application, and an annual fee of AED 50 for each toll free or premium rate number. If a specific toll free or premium rate number is requested the annual fee is AED 150 instead of AED 50. There are no categories of special numbers.

3.1.4 Short codes

TRA allocates short codes individually. There is an application fee of AED 500 for each application. There are no categories of special numbers.

3.2 Australia

3.2.1 Background

The regulator is the Australian Communications and Media Authority (ACMA). The main service providers are Telstra, Optus (controlled by SingTel) and Vodafone.

In national dialling, fixed and mobile numbers are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '2', '3', '7' or '8' and having nine digits.
- Mobile numbers beginning with '4' and having nine digits.
- Toll free numbers beginning with '1800' and having ten digits.
- Local rate numbers beginning with '1300' and having ten digits.
- Local rate numbers beginning with '13' and having six digits.
- Premium rate numbers beginning with '1900' and having ten digits.
- Short codes for general services beginning with '1' and having three or four digits.

There are also text message and nomadic numbers (and other short codes).

AUD 1=AED 2.87.

3.2.2 Fixed and mobile numbers

ACMA allocates fixed numbers in blocks of 100 or 1,000 and mobile numbers in blocks of 100,000. Numbers are allocated administratively (in response to an application). There is an annual fee of approximately AUD 0.64 for each fixed or mobile number; the precise fee is determined by the ACMA annual revenue target. There are no categories of special numbers.

Fixed and mobile numbers may be ported. They may also be traded; the ACMA regulations, *Telecommunications Numbering Plan 1997 as amended*, prohibit service providers from changing the numbers of customers except under clearly defined conditions that do not include trading. They can be allocated to companies that specialise in trading, not in telecommunications.

Vodafone lets customers select special ('star') mobile numbers from lists with prices such as:

- AUD 275 for a gold number.
- AUD 50 for a silver number.

Table 6 shows the definitions of the special numbers by Vodafone. It is drawn from a description on the Vodafone web site. That description appears to omit obvious patterns (such as '***aaa*bb' and '***aa*bbb') and duplicate certain patterns (such as '***$a_0a_0a_1a_1a_2a_2$' and '***$a_2a_2a_1a_1a_0a_0$'). The patterns, constraints and capacities in Table 6 might therefore not be exactly those intended by Vodafone. The following points from the ordering and categorisation might have general interest:

- Numbers that include repetitions or clusters are mainly higher in the ordering and categorisation than numbers that include sequences.
- Numbers that intersperse repetitions with sequences which ascend or descend by one at each step ('***$ab_0ab_1ab_2$' and '***$ab_2ab_1ab_0$') are regarded as special.
- Numbers that repeat the digits of sequences which ascend or descend by one at each step ('***$a_0a_0a_1a_1a_2a_2$' and '***$a_2a_2a_1a_1a_0a_0$', '***$a_0a_1a_1a_2a_2a_2$' and '***$a_2a_2a_2a_1a_1a_0$') are regarded as special.

Table 6 Special mobile numbers as defined by one service provider in Australia

Category	Pattern	Capacity
Gold	***aaaaaa	10,000
	**aaaaaa*	9,000
	*aaaaaa**	9,000
	***ababab	90,000
	**ababab*	81,000
	*ababab**	81,000
	***aaaabb	90,000
	**aaaabb*	90,000
	*aaaabb**	90,000
	***aabbaa	89,100
	**aabbaa*	90,000
	*aabbaa**	89,100
	***aabbbb	89,100
	**aabbbb*	90,000
	*aabbbb**	89,100
	***aabbcc	712,800
	**aabbcc*	720,000
	*aabbcc**	665,200

	***aabaab	90,000
	**aabaab*	90,000
	*aabaab**	90,000
	***abaaba	90,000
	**abaaba*	89,100
	*abaaba**	89,100
	***abbabb	90,000
	**abbabb*	90,000
	*abbabb**	90,000
	***abcabc	720,000
	**abcabc*	648,000
	*abcabc**	648,000
	****aaaaa	90,000
	***aaaaa*	81,000
	aaaaa	81,000
	*aaaaa***	90,000
	*****aaaa	900,000
	****aaaa*	810,000
	***aaaa**	801,000
	aaaa*	801,000
	*aaaa****	890,000
	***$a_0a_1a_2a_0a_1a_2$	0
	**$a_0a_1a_2a_0a_1a_2$*	0
	*$a_0a_1a_2a_0a_1a_2$**	0
	***$a_2a_1a_0a_2a_1a_0$	0
	**$a_2a_1a_0a_2a_1a_0$*	0
	*$a_2a_1a_0a_2a_1a_0$**	0
	***$a_0a_1a_1a_2a_2a_2$	7,200
	***$a_2a_2a_2a_1a_1a_0$	7,200
	***$a_0a_0a_1a_1a_2a_2$	0
	***$a_2a_2a_1a_1a_0a_0$	0
	***$ab_0ab_1ab_2$	80,000
	***$ab_2ab_1ab_0$	80,000
Silver	****aaabb	810,000
	****aabbb	875,000
	***aaabb*	810,000
	***aabbb*	793,800
	**aaa*bb*	648,000
	**aa*bbb*	648,000
	*aaa**bb*	656,100
	*aa**bbb*	656,100
	aaabb	656,100
	aabbb	729,000
	*aaa*bb**	648,000
	*aa*bbb**	648,000
	*aaabb***	729,000
	*aabbb***	72900

	*****aabb	7,290,000
	aabb*	6,561,000
	******aaa	8,100,000
	aaa**	7,290,000

Optus and Telstra do not appear to have any service offerings that distinguish special numbers from other numbers. However, until recently Telstra let customers select special mobile numbers from lists with prices such as:

- AUD 400 for a gold number.
- AUD 200 for a silver number.

Table 7 shows the definitions of the special numbers by Telstra. They differ in many details from those of Vodafone: some of the special numbers placed in a certain category by one might be placed in a different category, or in no category at all, by the other.

Table 7 Special mobile numbers as defined by another service provider in Australia

Category	Pattern	Capacity
Gold	***aaaaaa	10,000
	**aaaaaa*	9,000
	*aaaaaa**	9,000
	***ababab	90,000
	**ababab*	81,000
	*ababab**	81,000
	***aaabbb	90,000
	**aaabbb*	90,000
	*aaabbb**	90,000
	***aabaab	90,000
	**aabaab*	90,000
	*aabaab**	90,000
	***abaaba	90,000
	**abaaba*	89,100
	*abaaba**	89,100
	***abbabb	90,000
	**abbabb*	89,100
	*abbabb**	89,100
	***abcabc	720,000
	**abcabc*	648,000
	*abcabc**	648,000
	$***a_0a_1a_2a_3a_4a_5$	5,000
	$**a_0a_1a_2a_3a_4a_5*$	4,600
	$*a_0a_1a_2a_3a_4a_5**$	4,600
	$***a_5a_4a_3a_2a_1a_0$	5,000
	$**a_5a_4a_3a_2a_1a_0*$	4,600
	$*a_5a_4a_3a_2a_1a_0**$	4,600

Silver	****aaaaa	90,000
	***aaaaa*	81,000
	aaaaa	81,000
	*aaaaa***	90,000
	****aa*aa	801,000
	***aa*aa*	720,900
	aa*aa	720,900
	*aa*aa***	801,000
	***aaaabb	81,000
	**aaaabb*	72,900
	*aaaabb**	81,000
	***aabbaa	89,100
	**aabbaa*	90,000
	*aabbaa**	89,100
	***aabbbb	81,000
	**aabbbb*	81,000
	*aabbbb**	81,000
	***aabbcc	705,600
	**aabbcc*	720,000
	*aabbcc**	648,000
	***abbbba	81,000
	**abbbba*	81,000
	*abbbba**	81,000
	***abccba	720,000
	**abccba*	720,000
	*abccba**	720,000
	****$a_0a_1a_2a_3a_4$	55,000
	***$a_0a_1a_2a_3a_4$*	30,250
	$a_0a_1a_2a_3a_4$	30,250
	*$a_0a_1a_2a_3a_4$***	55,000
	****$a_4a_3a_2a_1a_0$	55,000
	***$a_4a_3a_2a_1a_0$*	30,250
	$a_4a_3a_2a_1a_0$	30,250
	*$a_4a_3a_2a_1a_0$***	55,000
	***$a_0a_1a_2$bbb	80,000
	***$a_2a_1a_0$bbb	80,000
	***aaa$b_0b_1b_2$	80,000
	***aaa$b_2b_1b_0$	80,000
	***$ab_0ab_1ab_2$	80,000
	***$ab_2ab_1ab_0$	80,000

The number trader MyNumber lets customers select special mobile numbers from lists with prices sampled in Table 8.

Table 8 Special mobile numbers as defined by one number trader in Australia

Category	Price	Pattern
Platinum	AUD 5,000	*aaa**aaa *aaa*bbbb
	AUD 1,000	*aaa*aaa* *aaa*abbb *aaab*bbb *aaabcabc
	AUD 500	*aaab*abb *aaa**bba *aaa**bbb *aaabccbb
Gold	AUD 300	*aaaaa*** *aaa**aa* *aaaa*b*b *aaa**bba *aaab*bb* *aaabcbc* *aaab*bcc *aaa*bbcc
Silver	AUD 125	*aaaa**** *aaa***a* *aaab**b* *aaa*b*ab *aaa**b*b

The MyNumber lists also include numbers incorporating ‘138’ or ‘168’ and numbers matching words according to the key pad mapping of digits to letters.

There are many classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 9.

Table 9 Special mobile numbers as priced in classified advertisements in Australia

Price	Number
AUD 10,000	420202020
AUD 5,000	499999333
AUD 1,000	477798887
AUD 800	499941999
AUD 500	4**aaabbb
AUD 300	4aabbccdd

3.2.3 Toll free and premium rate numbers

ACMA allocates toll free, local rate and premium rate numbers individually. There is an annual fee of:

- AUD 0.64 for each ten-digit toll free, local rate or premium rate number.

- AUD 6,400 for each six-digit local rate number.

The fee stated here is approximate; the precise fee is determined by the ACMA annual revenue target and multiplied by ten for each reduction in number length by one digit.

In 2004, ACMA created five categories of special ('smart') toll free and local rate numbers (called 'platinum', 'diamond', 'gold', 'opal' and 'silver'). Some of these were released in an online auction every fortnight; approximately 250 numbers were sold in each auction. The numbers could also be obtained direct from a service provider on a "first-come first served" basis, or for emergency purposes. Between 2004 and 2010, over 30,000 numbers were sold by auction, with a return total payment of over AUD 32,000,000. Charities meeting complicated qualification standards received concessionary prices.

In 2010, ACMA reduced the reserve prices and abolished both the complicated qualification standards and the concessionary prices for charities. There were then reserve prices of:

- AUD 20,000 (reduced from AUD 40,000) for a ten-digit platinum number.
- AUD 16,000 (reduced from AUD 27,500) for a six-digit platinum number.
- AUD 8,000 (reduced from AUD 12,500) for a diamond number.
- AUD 6,000 (reduced from AUD 9,000) for a six-digit gold number.
- AUD 4,500 (reduced from AUD 12,500) for a ten-digit gold number.
- AUD 2,400 (reduced from AUD 3,000) for a six-digit opal number.
- AUD 1,500 (reduced from AUD 3,000) for a ten-digit opal number.
- AUD 1,200 (reduced from AUD 1,500) for a six-digit silver number.
- AUD 750 (reduced from AUD 2,500) for a ten-digit silver number.
- AUD 400 (reduced from AUD 500) for a six-digit other number.
- AUD 250 (reduced from AUD 500) for a ten-digit other number.

Even then platinum ten-digit numbers were less expensive than the corresponding six-digit ones before the annual fees are taken into account.

In 2015, ACMA abolished the auctions and allowed potential number holders to obtain numbers direct from ACMA on a "first-come first served" basis. However, ACMA did not abolish the extended rights of use associated with these special toll free and local rate numbers. These rights of use entitle the number holders to trade the numbers and to leave the numbers inactive for up to three years (after which they are reclaimed to avoid wastage from the number supply).

The diamond and opal numbers are ones that match words according to the key pad mapping of digits to letters: ten-digit diamond ones have word counts of at least seventeen, six-digit diamond ones have word counts of at least eight, ten-digit opal ones have word counts of at least thirteen and six-digit opal ones have word counts of at least six.

Table 10 shows the definitions of the special numbers by ACMA other than the diamond and opal numbers. It is drawn from a verbal description in the ACMA price list. That description appears to omit obvious patterns (such as ‘1300$a_0a_0a_1a_1a_2a_2$’ and ‘1800$a_0a_0a_1a_1a_2a_2$’) and make some dubious categorisations (by regarding ‘13*$a_0a_1a_2$’ and ‘13*$a_2a_1a_0$’ as only silver patterns, for example). The patterns, constraints and capacities in the table might therefore not be exactly those intended by ACMA. The following points from the ordering and categorisation might have general interest:

- Numbers that are uniform near their middles are higher in the ordering and categorisation than numbers that are uniform near their beginnings and ends only.
- Numbers that include sequences which ascend or descend by two at each step (‘13$a_0a_2a_4a_6$‘ and ‘13$a_6a_4a_2a_0$‘) are regarded as special, though not as special as numbers that include sequences which ascend or descend by one at each step (‘13$a_0a_1a_2a_3$‘ and ‘13$a_3a_2a_1a_0$’).
- Numbers that repeat the digits of sequences which ascend or descend by one at each step (‘1300$a_2a_2a_1a_1a_0a_0$‘ and ‘1800$a_2a_2a_1a_1a_0a_0$’, ‘1300$a_0a_0a_1a_1a_2a_2$’ and ‘1800$a_0a_0a_1a_1a_2a_2$‘) are regarded as special, and the descending ones are regarded as more special than the ascending ones.

Table 10 Special toll free and local rate numbers as defined by the regulator in Australia

Category	Pattern	Capacity
Platinum	1300aaaaaa	10
	1800aaaaaa	10
	1300aaabbb	90
	1800aaabbb	90
	1300ababab	90
	1800ababab	90
	1300abbbba	90
	1800abbbba	90
	1300$a_0a_1a_2a_3a_4a_5$	5
	1800$a_0a_1a_2a_3a_4a_5$	5
	1300$a_5a_4a_3a_2a_1a_0$	5
	1800$a_5a_4a_3a_2a_1a_0$	5
	1300$a_2a_2a_1a_1a_0a_0$	5
	1800$a_2a_2a_1a_1a_0a_0$	5
	13aaaa with $a \neq 0$	9

	13aabb with $a \neq 0$	81
	13abab with $a \neq 0$	81
	13abba with $a \neq 0$	81
	13$a_0a_1a_2a_3$ with $a_0 \neq 0$	6
	13$a_3a_2a_1a_0$	7
	1313**	99
	13a13* with $a \neq 0$	88
	13a*13 with $a \neq 0$	88
Gold	1300aaaaa* 1800aaaaa*	90 90
	1300aaaabb 1800aaaabb	90 90
	1300aaaa** 1800aaaa**	810 810
	1300aabbbb 1800aabbbb	90 90
	1300$a_0a_1a_2$bbb 1800$a_0a_1a_2$bbb	80 80
	1300$a_2a_1a_0$bbb 1800$a_2a_1a_0$bbb	80 80
	1300bbb$a_0a_1a_2$ 1800bbb$a_0a_1a_2$	80 80
	1300bbb$a_2a_1a_0$ 1800bbb$a_2a_1a_0$	80 80
	1300aabbaa 1800aabbaa	90 90
	1300aabbcc 1800aabbcc	720 720
	1300$a_0a_0a_1a_1a_2a_2$ 1800$a_0a_0a_1a_1a_2a_2$	8 8
	13001300** 18001800**	90 90
	13abbb with $a \neq 0$ and $a \neq 1$	80
	13aaab with $a \neq 0$ and $a \neq 1$	80
	13abbb with $a=1$ and $b \neq 3$	8
	13aaab with $a=1$ and $b \neq 3$	8
	13$a_0a_1a_2$* with $a_0 \neq 0$	64
	13$a_2a_1a_0$*	73
Silver	1300*aaaaa 1800*aaaaa	90 90
	1300**aaaa 1800**aaaa	810 810
	1300aaa*** 1800aaa***	8,910 8,910
	1300$a_0a_1a_2a_2a_1a_0$ 1800$a_0a_1a_2a_2a_1a_0$	8 8
	1300$a_2a_1a_0a_0a_1a_2$	8

	$1800a_2a_1a_0a_0a_1a_2$	8
	$1300a_0a_1bba_1a_0$ $1800a_0a_1bba_1a_0$	82 82
	$1300a_1a_0bba_0a_1$ $1800a_1a_0bba_0a_1$	82 82
	$1300ab_0b_1b_1b_0a$ $1800ab_0b_1b_1b_0a$	82 82
	$1300ab_1b_0b_0b_1a$ $1800ab_1b_0b_0b_1a$	82 82
	1300aabaab 1800aabaab	90 90
	1300abaaba 1800abaaba	90 90
	1300abcabc 1800abcabc	720 720
	1300abccba 1800abccba	720 720
	$1300a_0a_1a_2b_0b_1b_2$ $1800a_0a_1a_2b_0b_1b_2$	59 59
	1300**1300 1800**1800	100 100
	$13a_0a_2a_4a_6$ with $a_0 \neq 0$	3
	$13a_6a_4a_2a_0$	4
	$13ab_0b_1b_2$ with $a \neq 0$	66
	$13ab_2b_1b_0$ with $a \neq 0$	65
Other	1300aababa 1800aababa	90 90
	1300aabcbc 1800aabcbc	720 720
	1300abab** 1800abab**	8,910 8,910
	1300***aaa 1800***aaa	9,000 9,000
	1300**abab 1800**abab	8,820 8,820
	$1300ab_0b_1b_2b_3a$ $1800ab_0b_1b_2b_3a$	60 60
	$1300ab_3b_2b_1b_0a$ $1800ab_3b_2b_1b_0a$	60 60
	1300aa**aa 1800aa**aa	720 720
	13a*aa with $a \neq 0$	81
	13aa** with $a \neq 0$	712
	13a*bb	738
	13abb*	722

3.2.4 Short codes

ACMA allocates short codes individually. There are no fees. There are no categories of special numbers.

3.3 Bahrain

3.3.1 Background

The regulator is the Telecommunications Regulatory Authority (TRA). The main service providers are Batelco, Viva (controlled by Saudi Telecom) and Zain.

The numbering plan provides:

- Fixed numbers beginning with '1' and having eight digits.
- Mobile numbers beginning with '3' and having eight digits.
- Toll free numbers beginning with '800' and having eight digits.
- Premium rate numbers beginning with '900' and having eight digits.
- Short codes for general services beginning with '0', '1', '8' or '9' and having three or five digits.

BHD 1=AED 9.74.

3.3.2 Fixed and mobile numbers

TRA allocates fixed and mobile numbers in blocks of 10,000. There are an application fee of BHD 25 for each application, and an annual fee of BHD 0.1 for each fixed or mobile number. There are no categories of special numbers.

Fixed and mobile numbers may be ported. They may also be traded.

Batelco lets customers select special mobile numbers from lists with prices sampled in Table 11.

Table 11 Special mobile numbers as defined by one service provider in Bahrain

Category	Price	Pattern
	BDH 500	abaaaba* abaabab* abbaaab* abbabaa* abacccba* *a*abbbb

	BDH 400	abb*babb a*ababa* a*abbba* ****aaaa abacbacb abbaccdd
	BDH 300	abaab*aa abaaba** a**bbbba **abaaba abaacabc ababccc*
	BDH 200	***aaaa* abcacba abccc*ca **aabbcc *abbbacc abcbccdd
	BDH 100	a*aaa*** **a*aaa* *a*aabbb **aa*bbb *a*bbabb a*abcbbc
	BDH 50	*a*aabbb *a*bbabb abccbc*a abccbba* abccbadd abcbcdda
	BDH 5 (reduced from BDH 100)	a*aaabba* a*ababb* abaabcc* abaccadd
	BDH 0	abaab*aa abba*ab* ab**bbb* *a*aa*bb abbca*c* abc*bbca

Viva offers customers three categories of special ('premium') mobile numbers (called 'platinum', 'gold' and silver'), with certain minimum prices:

- BDH 500 for a platinum number.
- BDH 100 for a gold number.

Number management practices

- BDH 50 for a silver number.

Viva also includes special numbers in particular tariff packages.

Zain includes special numbers in particular tariff packages.

There are some classified advertisements selling special mobile numbers. At the time of writing '34060560' was on offer for BHD 600.

3.3.3 Toll free and premium rate numbers

TRA allocates toll free and premium rate numbers in blocks of 1,000. There are an application fee of BHD 25 for each application, and an annual fee of BHD 0.1 for each toll free or premium rate number. There are no categories of special numbers.

3.3.4 Short codes

TRA allocates short codes individually. There are an application fee of BHD 25 for each application and an annual fee of:

- BHD 10,000 for a three-digit short code.
- BHD 1,000 for a four-digit short code.
- BHD 100 for a five-digit short code.

There are no categories of special numbers.

3.4 Belgium

3.4.1 Background

The regulator is the Belgian Institute for Postal services and Telecommunications (BIPT). The main service providers are Belgacom, Mobistar and KPN.

In national dialling, fixed and mobile numbers, and toll free and premium rate numbers, are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '1', '2', '3', '4', '5', '6', '7', '8' or '9' and having eight digits.
- Mobile numbers beginning with '4' or '7' and having nine digits.
- Toll free numbers beginning with '800' and having nine digits.
- Premium rate numbers beginning with '70' and having eight digits.
- Premium rate numbers beginning with '90' and having nine digits.
- Short codes for general services beginning with '1' and having three, four, five or six digits.

- Short codes for text messages beginning with '2', '3', '4', '5', '6', '7', '8' or '9' and having four digits.

There are also local rate, machine-to-machine and virtual private network numbers.

EUR 1=AED 4.11.

3.4.2 Fixed and mobile numbers

BIPT allocates fixed numbers in blocks of 1,000 or 10,000 and mobile numbers in blocks of 100,000. There is an application fee of EUR 30 for each application for fixed numbers and EUR 1,167 for each application for mobile numbers, and an annual fee of EUR 59 for each block of 1,000 fixed numbers, EUR 117 for each block of 10,000 fixed numbers and EUR 1,750 for each block of mobile numbers. There are no categories of special numbers.

Fixed and mobile numbers can be ported. Number transfers between service providers require approval by BIPT.

Belgacom, Mobistar and KPN do not appear to have any service offerings that distinguish special numbers from other numbers.

Mobistar lets customers select their own mobile numbers for EUR 27.95. In addition it lets customers enter four digits of their favoured mobile numbers to select numbers without extra charge.

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 12.

Table 12 Special mobile numbers as priced in classified advertisements in Belgium

Price	Number
EUR 250	487676676
EUR 129	484666697
EUR 60	47*565454

3.4.3 Toll free and premium rate numbers

BIPT allocates toll free numbers in blocks of 1,000 and premium rate numbers in blocks of 1,000 or 10,000. There is an application fee of EUR 1,167 for each application, and an annual fee of EUR 875 for each block of toll free numbers, EUR 875 for each block of 1,000 premium rate numbers and EUR 1,750 for each block of 10,000 premium rate numbers. There are no categories of special numbers.

Toll free and premium rate numbers can be ported.

3.4.4 Short codes

BIPT allocates short codes individually. There is an application fee of EUR 1,167 for each application, and an annual fee of EUR 5,834 or EUR 14,584 (depending on the second digit), for each short code for general services. There is an application fee of EUR 20 for each application and an annual fee of EUR 10, EUR 39 or EUR 97 (depending on the first digit), for each short code for text messages. There are no categories of special numbers.

3.5 Egypt

3.5.1 Background

The regulator is the National Telecommunications Regulatory Authority (NTRA). The main service providers are Etisalat, Mobinil (controlled by Orange) and Vodafone.

In national dialling, mobile numbers, and toll free and premium rate numbers, are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '1', '2', '3', '4', '5', '6', '8' or '9' and having eight or nine digits.
- Mobile numbers beginning with '1' and having ten digits.
- Toll free numbers beginning with '800' and having ten digits.
- Premium rate numbers beginning with '900' and having eight digits.
- Short codes for general services beginning with '1' and having three or five digits.

There are also internet access numbers.

EGP 1=AED 0.48.

3.5.2 Fixed and mobile numbers

NTRA allocates fixed numbers in blocks of 10,000 and mobile numbers in blocks of 100,000. No fees are known. There are no categories of special numbers.

Mobile numbers may be ported. However, they must not be transferred or traded except with permission from NTRA.

Vodafone offers special numbers at prices between EGP 250 and EGP 20,000, with reductions for employees. Available prices at the time of writing are exhibited in Table 13.

Table 13 Special mobile numbers as priced by one service provider in Egypt

Price	Number
EGP 2,500	1099992272
EGP 1,250	1099991779
EGP 500	1025077772
EGP 250	1020006620

Vodafone also includes special numbers in particular tariff packages.

Mobinil includes special numbers in particular tariff packages.

Etisalat does not appear to have any service offerings that distinguish special numbers from other numbers.

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 14.

Table 14 Special mobile numbers as priced in classified advertisements in Egypt

Price	Number
EGP 16,000	106*333333
EGP 11,000	11555555*6
EGP 1,200	1124111125

3.5.3 Toll free and premium rate numbers

NTRA allocates toll free and premium rate numbers individually. No fees are known.

3.5.4 Short codes

NTRA allocates short codes individually. No fees are known.

3.6 France

3.6.1 Background

The regulator is the Autorité de Régulation des Communications Électroniques et des Postes (ARCEP). The main service providers are Orange, Bouygues, Iliad and SFR (controlled by Numéricable).

In national dialling, fixed and mobile numbers, and toll free and premium rate numbers, are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '1', '2', '3', '4' or '5' and having nine digits.
- Mobile numbers beginning with '6' or '7' and having nine digits.
- Toll free numbers beginning with '80' and having eight digits.

- Premium rate numbers beginning with '89' and having eight digits.
- Short codes for general services beginning with '1' or '3' and having four or six digits.

There are also virtual private network numbers. Short codes for text messages are outside the numbering plan.

EUR 1=AED 4.11.

3.6.2 Fixed and mobile numbers

ARCEP allocates fixed numbers in blocks of 10,000 and mobile numbers in blocks of 10,000 or 100,000. There is an annual fee of EUR 0.02 for each fixed or mobile number. There are no categories of special numbers.

Fixed and mobile numbers can be ported. Number transfers between service providers require approval by ARCEP.

Orange, Bouygues, Iliad and SFR do not appear to have any service offerings that distinguish special numbers from other numbers.

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 15.

Table 15 Special mobile numbers as priced in classified advertisements in France

Price	Number
EUR 1,000	603366366
EUR 950	771575757
EUR 490	652687687
EUR 490	632424344
EUR 50	783839492
EUR 30	758845898

3.6.3 Toll free and premium rate numbers

ARCEP allocates toll free and premium rate numbers in blocks of 1,000. There is an annual fee of EUR 0.02 for each toll free or premium rate number.

Toll free and premium rate numbers can be ported.

3.6.4 Short codes

ARCEP allocates short codes individually. There is an annual fee of EUR 40,000 for each short code.

SMS+, an association that is run by the mobile service providers and is now incorporated in the Association Française du Multimédia Mobile (AFMM) manages the distribution of text message short codes. It has four categories of special short codes (called 'gold', 'silver', 'bronze' and 'nickel'). Different mobile service providers charge value added service providers different amounts for selecting short codes in these categories; they also charge different amounts if they make the selections for the value added service providers.:

Table 16 shows the definitions of the special short codes by SMS+. It is drawn from the description by the mobile service providers. That description varied an earlier suggestion, especially by excluding patterns with descending sequences (such as '*$a_3a_2a_1a_0$' and '**$a_2a_1a_0$') and patterns to continue the analogy between gold and silver numbers into the bronze ones (such as '***aa' and '**a_0a_1'). It involves the use of a particular digit ('0'), repetitions, and sequences.

Table 16 Special short codes for text messages as defined by a mobile service provider association in France

Category	Pattern	Capacity
Gold	aaaaa with a≥3 and a≤8	6
	abbbb with a≥3 and a≤8 and b≥1	48
	$ab_0b_1b_2b_3$ with a≥3 and a≤8 and b_0≥1	36
	aa000 with a≥3 and a≤8 and b≥1	6
	ab000 with a≥3 and a≤8 and b≥1	48
Silver	aabbb with a≥3 and a≤8 and b≥1	48
	abaaa with a≥3 and a≤8 and b≥1	48
	abccc with a≥3 and a≤8 and b≥1 and c≥1	336
	aa012 with a≥3 and a≤8	6
	ab012 with a≥3 and a≤8 and b≥1	48
	$aac_1c_2c_3$ with a≥3 and a≤8 and c_0≠a	38
	$abc_1c_2c_3$ with a≥3 and a≤8 and b≥1 and c_0≠b	304
	aaa00 with a≥3 and a≤8	6
	aab00 with a≥3 and a≤8 and b≥1	48
	aba00 with a≥3 and a≤8 and b≥1	48
	abb00 with a≥3 and a≤8 and b≥1	48
	abc00 with a≥3 and a≤8 and b≥1 and c≥1	336
Bronze	aaabb with a≥3 and a≤8 and b≥1	48
	abbaa with a≥3 and a≤8 and b≥1	48
	abbcc with a≥3 and a≤8 and b≥1 and c≥1	336
	aabab with a≥3 and a≤8 and b≥1	48
	ababa with a≥3 and a≤8 and b≥1	48
	abcbc with a≥3 and a≤8 and b≥1 and c≥1	336
	abaab with a≥3 and a≤8 and b≥1	48
	abbab with a≥3 and a≤8 and b≥1	48
	abcab with a≥3 and a≤8 and b≥1 and c≥1	336

	$ab_0b_1b_0b_2$ with $a\geq3$ and $a\leq8$ and $b_0\geq1$	42
	aa0a0 with $a\geq3$ and $a\leq8$	6
	aa0b0 with $a\geq3$ and $a\leq8$ and $b\geq1$	48
	ab0b0 with $a\geq3$ and $a\leq8$ and $b\geq1$	48
	ab0c0 with $a\geq3$ and $a\leq8$ and $b\geq1$ and $c\geq1$	336
Nickel	aa*** with $a\geq3$ and $a\leq8$	5,574
	ab*** with $a\geq3$ and $a\leq8$ and $b\geq1$	44,592

3.7 Hong Kong

3.7.1 Background

The regulator is the Office of the Communications Authority (OFCA), which succeeded the Office of the Telecommunications Authority (OFTA). The main service providers are Hutchison, Hong Kong Telecommunications and SmarTone.

The numbering plan provides:

- Fixed numbers beginning with '2' or '3' and having eight digits.
- Mobile numbers beginning with '5', '6' or '9' and having eight digits.
- Toll free numbers beginning with '800' and having nine digits.
- Premium rate numbers beginning with '900' and having eight or eleven digits.
- Short codes for general services beginning with '0' or 1' and having three, four, five, six or seven digits.
- Short codes for emergency services beginning with '99' and having three digits.

There are also calling card, text message, personal and paging numbers.

HKD 1=AED 0.47.

3.7.2 Fixed and mobile numbers

OFCA allocates fixed and mobile numbers in blocks of 100,000. There is an annual fee of HKD 3 for each fixed or mobile number.

Fixed and mobile numbers may be ported. They may also be traded.

In 1997, OFTA issued a consultation document about introducing special numbers. Whether to do this remained under review. In 2002, OFTA issued a revised version of the consultation document that enlarged the two categories of special numbers (called 'primary' and 'secondary'). Primary numbers would have been sold in auctions, with a reserve price of HKD 10,000 for each special number; secondary numbers would have been allocated administratively (in response to an application), with a standard fee between HKD 200 and HKD 500 for each special number. In addition, consumers could pay the same standard fee to be assigned numbers of their choice. The proceeds would be devoted to education or research in telecommunications. OFTA ultimately decided not to do this but instead to continue to allocate numbers administratively (in response to an application). Nonetheless the numbering plan includes special number blocks, which are specified by their leading three digits and intended for allocation to charities. There is no special fee for a special number.

Table 17 shows the definitions of the special numbers by OFTA on which there was a consultation. It is drawn from an enumeration in the 2002 consultation document but ignores some constraints on the initial and subsequent digits. That enumeration extended the one in the 1997 consultation document, especially by including patterns with three contiguous repeated digits (such as '*****aaa' and '****aaa*') and patterns containing specific digits (such as '****8a88' and '****88a8'). The following points from the ordering and categorisation might have general interest:

- Numbers that are uniform near their ends only are regarded as special, though not as special as numbers that are uniform near their beginnings as well.
- Numbers that include sequences which first ascend by one at each step and then repeat two of the digits from the sequence ('$a_0a_1a_2a_3a_3a_1a_1a_1$' and '$a_0a_1a_2a_3a_0a_0a_2a_2$') are regarded as special.
- Numbers that repeat the digits of sequences which ascend or descend by one at each step ('$a_3a_3a_2a_2a_1a_1a_0a_0$' and '$a_0a_0a_1a_1a_2a_2a_3a_3$') are regarded as special.

Table 17 Special fixed and mobile numbers as earlier defined by the regulator in Hong Kong

Category	Pattern	Capacity
Primary	aaaaaaaa with a≥2 and a≤8	7
	aaaabbbb with a≥2 and a≤8	63
	aabbaabb with a≥2 and a≤8	63
	aabbbbaa with a≥2 and a≤8	63
	abababab with a≥2 and a≤8	63
	ababbbba with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b=2	38

	abababbb with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b≥2 and b≤3 or a=9 and b≥5 and b≤8	43
	ababaaab with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b≥2 and b≤3 or a=9 and b≥5 and b≤8	43
	ababbaaa with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b≥2 and b≤3 or a=9 and b≥5 and b≤8	43
	ababbbaa with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b≥2 and b≤3 or a=9 and b≥5 and b≤8	43
	ababaabb with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b≥2 and b≤3 or a=9 and b≥5 and b≤8	43
	ababbbbb with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b≥2 and b≤3 or a=9 and b≥5 and b≤8	43
	ababaaaa with a=2 and b=0 or a=3 or a≥6 and a≤8 or a=9 and b≥2 and b≤3 or a=9 and b≥5 and b≤8	43
	ababbaab with a=2 and b=0 or a=3 or a≥6 and a≤8	37
	23456789	1
	34567890	1
	$a_7a_6a_5a_4a_3a_2a_1a_0$	3
	$a_0a_0a_1a_1a_2a_2a_3a_3$ with $a_0 \geq 2$	5
	$a_3a_3a_2a_2a_1a_1a_0a_0$ with $a_0 \leq 5$	6

	$a_0a_1a_2a_3a_0a_0a_1a_1$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_1a_1a_2a_2$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_2a_2a_3a_3$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_0a_0a_2a_2$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_1a_1a_3a_3$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_0a_0a_3a_3$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_1a_1a_0a_0$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_2a_2a_1a_1$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_3a_3a_2a_2$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_2a_2a_0a_0$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_3a_3a_1a_1$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_3a_3a_0a_0$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_0a_1a_1a_1$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_1a_2a_2a_2$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_2a_3a_3a_3$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_1a_3a_3a_3$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_0a_3a_3a_3$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_1a_0a_0a_0$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_2a_1a_1a_1$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_3a_2a_2a_2$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_2a_0a_0a_0$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_3a_1a_1a_1$ with $a_0=3$ or $a_0=6$ $a_0a_1a_2a_3a_3a_0a_0a_0$ with $a_0=3$ or $a_0=6$	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
	aaaabbcc with (a≥2 and a≤3 or a≥6 and a≤7) and (b≥2 and b≤3 or b≥6 and b≠7) and (c≥2 and c≤3 or c≥6 and c≠7)	97
Secondary	*****aaa	899,914
	****aaa*	899,919
	****aabb	999,868
	****abab	899,937
	****abba	899,963
	****$a_0a_1a_2a_3$	69,999
	****$a_3a_2a_1a_0$	69,997
	****0789	10,000
	****7890	9,999
	****8a88 with a≠5 and a≠8	80,000
	****88a8 with a≠5 and a≠8	80,000
	****1223 ****1323 ****1623	10,000 10,000 10,000

	****1228	10,000
	****1328	10,000
	****1628	10,000
	****3328	10,000
	****8328	10,000
	****9228	10,000
	****9328	10,000
	****1938	10,000
	****2238	10,000
	****2338	10,000
	****2638	10,000
	****2838	10,000
	****6338	10,000
	****6638	10,000
	****8238	10,000
	****9838	10,000
	****1168	10,000
	****1368	10,000
	****1668	10,000
	****1868	10,000
	****2268	10,000
	****3668	10,000
	****6268	10,000
	****1288	10,000
	****1388	10,000
	****1688	10,000
	****1788	10,000
	****1888	10,000
	****2388	10,000
	****2688	10,000
	****3288	10,000
	****6988	10,000
	****6833	10,000
	****8663	10,000
	****9899	10,000

Table 18 shows the definitions of the special numbers by OFTA. It is drawn from an enumeration in the numbering plan. That enumeration is much briefer than the one in the 2002 consultation.

Table 18 Special fixed and mobile numbers as later defined by the regulator in Hong Kong

Category	Pattern	Capacity
	aaa***** with a≥2 and a≤8	700,000
	$a_0a_1a_2$***** with a_0≥2	600,000
	890*****	100,000
	aba***** with a≥2	7,200,000

Hutchison, Hong Kong Telecommunications and SmarTone do not appear to have any service offerings that distinguish special numbers from other numbers.

There are some classified advertisements selling special mobile numbers. At the time of writing '98388588' was on offer for HKD 1,680,000. There are also many street booths selling special mobile numbers for prices typically between HKD 100 and HKD 10,000. In 2010, OFTA commissioned a consumer survey, *Survey on Demand of Special Telecommunications Numbers*, on the method of assigning special numbers; it concluded that there was little dissatisfaction with the existing method and limited interest in obtaining special numbers in auctions.

3.7.3 Toll free and premium rate numbers

OFCA allocates toll free and premium rate numbers in blocks of 1,000. There is an annual fee of HKD 3 for each toll free or premium rate number. There are no categories of special numbers.

3.7.4 Short codes

OFCA allocates short codes individually. There are no fees. There are no categories of special numbers.

3.8 Ireland

3.8.1 Background

The regulator is the Commission for Communications Regulation (Comreg). The main service providers are Eircom, Vodafone and 3 (controlled by Hutchison).

In national dialling, fixed and mobile numbers are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '1', '2', '4', '5', '6', '7' or '9' and having seven, eight or nine digits.
- Mobile numbers beginning with '8' and having nine digits.
- Toll free numbers beginning with '1800' and having ten digits.

- Premium rate numbers beginning with '15' and having four digits.
- Short codes for general services beginning with '1' and having three, four, five or six digits.
- '999' for emergency services.
- Short codes for text messages beginning with '5' and having five digits.

There are also nomadic, shared cost, shared revenue, personal and universal access numbers.

EUR 1=AED 4.11.

3.8.2 Fixed and mobile numbers

Comreg allocates fixed numbers in blocks of 1,000 and mobile numbers in blocks of 100,000. There are no fees. There are no categories of special numbers.

Fixed and mobile numbers may be ported. However, they must not be traded; the Comreg regulations, *National Numbering Conventions*, remark that trading numbers will be seen as evidence that the original holder does not require the numbers and will be considered to justify immediate recovery of the numbers.

Eircom, Vodafone and 3 do not appear to have any service offerings that distinguish special numbers from other numbers.

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 19.

Table 19 Special mobile numbers as priced in classified advertisements in Ireland

Price	Number
EUR 100	868672333
EUR 100	8*2271666
EUR 75	8*2288442
EUR 60	852010666
EUR 50	851010706
EUR 50	868600027

3.8.3 Toll free and premium rate numbers

Comreg allocates toll free and premium rate numbers individually. There are no fees. There are no categories of special numbers.

Toll free and premium rate numbers may be ported.

3.8.4 Short codes

Comreg allocates short codes individually. There are no fees. There are no categories of special numbers.

3.9 Kuwait

3.9.1 Background

The regulator is the Ministry of Communications, which is also the only fixed network service provider. In 2014/1435, a bill to institute the Telecommunications Regulatory Authority (TRA).led to the appointment of the TRA board. The main service providers are Ooredoo, Viva (controlled by Saudi Telecom) and Zain.

The numbering plan provides:

- Fixed numbers beginning with '1' and having seven digits.
- Fixed numbers beginning with '2' and having eight digits.
- Mobile numbers beginning with '5', '6' or '9' and having eight digits.
- Short codes for general services beginning with '1' and having three digits.
- '777' for emergency services.
- Short codes for text messages beginning with '5' or '9' and having three or five digits.

KWD 1=AED 12.41.

3.9.2 Fixed and mobile numbers

The Ministry of Communications allocates fixed and mobile numbers in blocks of 10,000. No fees are known. There are no categories of special numbers.There are no prohibitions on trading numbers.

Viva includes special numbers in particular tariff packages.

Ooredoo and Zain do not appear to have any service offerings that distinguish special numbers from other numbers.

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 20.

Table 20 Special mobile numbers as priced in classified advertisements in Kuwait

Price	Number
KWD 350	99222**2
KWD 260	55514040
KWD 150	55766167
KWD 65	98080335
KWD 50	51551558
KWD 30	55998696

3.9.3 Toll free and premium rate numbers

There are no toll free or premium rate numbers.

3.9.4 Short codes

The Ministry of Communications allocates short codes individually. No fees are known. There are no categories of special numbers.

3.10 Lebanon

3.10.1 Background

The regulator is the Telecommunications Regulatory Authority (TRA). In 2009/1430, TRA drafted a new numbering plan and a new numbering regulation; however, the Ministry of Telecommunications is still following the old numbering plan, so the fixed network continues to occupy the numbering space and fees proposed by TRA are not yet in place. The main service providers are Alfa (managed by Orascom) and Touch (managed by Zain).

The numbering plan provides:

- Fixed numbers beginning with '1', '4', '5', '6', '7' '8' or '9' and having seven digits.
- Mobile numbers beginning with '3' or '7' and having eight digits.
- Toll free numbers beginning with '80' or '81' and having eight digits.
- Premium rate numbers beginning with '90' or '91' and having eight digits.
- Short codes for general services beginning with '1' and having three or four digits.

LBP 1=AED 0.0024; LBP 1=USD 0.00067.

3.10.2 Fixed and mobile numbers

TRA allocates fixed numbers in blocks of 10,000 and mobile numbers in blocks of 10,000 or 100,000. There are an allocation fee of LBP 225 for each fixed or mobile number, and an annual fee of LBP 112.50 for each fixed number and LBP 225 for each mobile number. There are no categories of special numbers.

Alfa lets customers select special mobile numbers from lists without extra charge, though a postpaid customer can pay an extra USD 110 (LBP 165,935) to obtain a 'bronze' number.

Touch lets customers enter eight digits of their favoured mobile numbers to obtain lists from which they can select numbers without extra charge.

There are many classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 21.

Table 21 Special mobile numbers as priced in classified advertisements in Lebanon

Price	Number
USD 9,000	71177177
USD 4,000	76882222
USD 3,000	76766766
USD 1,500	71577477
USD 682	70001682
USD 500	70970989
USD 275	71183193
USD 175	70939444
USD 80	76069089
USD 50	70877047

3.10.3 Toll free and premium rate numbers

TRA allocates toll free and premium rate numbers in blocks of 1,000. There are an allocation fee of LBP 225 for each toll free or premium rate number, and an annual fee of LBP 225 for each toll free or premium rate number.

3.10.4 Short codes

TRA allocates short codes individually. There is an annual fee of LBP 225,000 for each short code.

3.11 Norway

3.11.1 Background

The regulator is the Nasjonal kommunikasjonsmyndighet (Nkom). The main service providers are Telenor, TeliaSonera and Tele2.

The numbering plan provides:

- Fixed numbers beginning with '2', '3', '5', '6' or '7' and having eight digits.
- Mobile numbers beginning with '4' or '9' and having eight digits.
- Toll free numbers beginning with '80' and having eight digits.
- Premium rate numbers beginning with '82' and having eight digits.
- Short codes for general services beginning with '1' and having three, four or six digits.
- Short codes for general services and text messages beginning with '0' and having five digits.

There are also machine-to-machine and personal numbers.

NOK 1=AED 0.48.

3.11.2 Fixed and mobile numbers

Nkom allocates fixed numbers in blocks of 1,000 or 10,000 and mobile numbers in blocks of 10,000. There is an annual fee of NOK 0.03 for each fixed number and NOK 0.10 for each mobile number. There are no categories of special numbers.

Fixed and mobile numbers can be ported. Number transfers between service providers require approval by Nkom.

Telenor and TeliaSonera let customers select special mobile numbers from lists with prices such as:

- NOK 2,500 for a gold number.
- NOK 1,500 for a silver number.

Telenor lets customers select their own mobile numbers (other than gold and silver ones) for NOK 500.

Tele2 does not appear to have any service offerings that distinguish special numbers from other numbers.

The number trader Gullnummer lets customers select special mobile numbers from lists with prices sampled in Table 22. The prices given here for class A numbers are annual prices, for use during one year, not for permanent allocation; in fact the numbers can be rented for use during one month.

Table 22 Special mobile numbers as defined by one number trader in Norway

Category	Price	Pattern
Class A	NOK 47,880	aab00000 aba00000 a00b0000
Class B	NOK 37,790 (reduced from NOK 49,990)	aaabbbbb
	NOK 19,990	a**a0000
	NOK 14,790 (reduced from NOK 19,990)	****0000
Class C	NOK 14,990	**a00a00
	NOK 11,990	a0*0*0a0 abacacab
	NOK 9,990	*0*0*0*0 ***aaaaa $*a_0a_1a_2a_3a_4a_5a_6$

		a1b1a1a1 a1b1b1a1
	NOK 8,990	**$a_5a_4a_3a_2a_1a_0$ abaabbaa a1b0a1a0 a1b0a1a1 a1bba1a1
	NOK 7,990	a*a*a000 *$a_0a_1a_1a_2$000 *$a_0a_1a_2$ $a_2$000 a*abb000 *1a1a000 a1b1a1b1 $ab_0b_1b_2b_1b_2ab_0$ $ab_0b_1b_2ab_0b_1b_2$
	NOK 6,990	a*a**000 *1*1*000
	NOK 5,990	$ab_0b_1b_2$a1a1 abaababb a0ba000b
	NOK 4,990	*$a_0a_1a_2a_3$000
	NOK 3,990	a**000a0 ***0*000
Class D	NOK 4,990	aa*a0a00 *a*a0a00 ***000*0
	NOK 3,990	*a**aaaa *a*aaa00 **aa0a00 **a00a00 **aabb00 *abbbaaa *abbaabb *ababa00
	NOK 3,490	*a*a0*00 *a*0aa00 **aa0*00 **aaabbb **aabbbb *abaab00 *ababb00 *abbba00
	NOK 2,990	a0**00a0 **aaa*00 *$a_0a_1a_1a_2a_2a_3a_3$ *$a_0a_1a_2a_2a_2$00
	NOK 2,490	a*aa**00 **0***00

		*1*1*100 $ab_0b_1b_2aa00$
	NOK 1,990	*aa***00 *a*a**00 **a*a*00 ***a*a00 a*a*bb00 ab*a*b00

The Gullnummer lists also include numbers matching words according to the key pad mapping of digits to letters.

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 23.

Table 23 Special mobile numbers as priced in classified advertisements in Norway

Price	Number
NOK 9,900	48444844
NOK 7,900	48266666
NOK 6,500	94121212
NOK 1,500	48444666
NOK 1,500	48444422
NOK 200	45449496

3.11.3 Toll free and premium rate numbers

Nkom allocates toll free and premium rate numbers in blocks of 100. There is an annual fee of NOK 0.15 for each toll free or premium rate number.

Toll free and premium rate numbers can be ported.

3.11.4 Short codes

Nkom allocates short codes individually. There is an annual fee of:

- NOK 0 for a three-digit short code.
- NOK 3,000 for a four-digit short code.
- NOK 2,000 for a six-digit short code.

There are, however, different fees for the five-digit-short codes beginning with ‘0’, which are in six categories. There is an annual fee of:

- NOK 123,700 for a class A five-digit short code.
- NOK 88,700 for a class B five-digit short code.
- NOK 68,700 for a class C five-digit short code.

- NOK 48,700 for a class D five-digit short code.
- NOK 23,700 for a class E five-digit short code.
- NOK 4,700 for a class F five-digit short code.

Table 24 shows the definitions of the special short codes by Nkom. It is drawn from the description in the Nkom statement. That description varied an earlier one, especially by splitting the bottom category in two. It involves the use of a particular digit ('0'), repetitions, and sequences.

Table 24 Special short codes as defined by the regulator in Norway

Category	Pattern	Capacity
Class A	0aaaa with a≥2	8
	0a000 with a≥2	8
Class B	0aaa0 with a≥2	8
	0aa00 with a≥2	8
	0a0a0 with a≥2	8
	0a00a with a≥2	8
Class C	0aabb with a≥2 and b≠0	64
	0abab with a≥2 and b≠0	64
	0abbb with a≥2 and b≠0	64
	0ab00 with a≥2 and b≠0	64
Class D	$0a_0a_1a_2a_3$ with $a_0 \geq 2$	5
	$0a_3a_2a_1a_0$ with $a_0 \geq 2$	5
	$0a_0a_2a_4a_6$ with $a_0 \geq 2$	2
	$0a_0a_1a_20$ with $a_0 \geq 2$	6
	$0a_0a_3a_60$ with $a_0 \geq 2$	1
	0abba with a≥2 and b≠0	64
	$0ab_0ab_1$ with $a \geq 2$	72
	$0a_09a_10$ with $a_0 \geq 2$	7
Class E	0ab*b with a≥2 and b≠0	648
	0a**0 with a≥2	759
Class F	0a*** with a≥2	6,127

3.12 Oman

3.12.1 Background

The regulator is the Telecommunications Regulatory Authority (TRA). The main service providers are Omantel and Ooredoo,

In national dialling, fixed and mobile numbers are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '2' and having eight digits.
- Mobile numbers beginning with '9' and having eight digits.

- Toll free numbers beginning with '800' and having eight digits.
- Premium rate numbers beginning with '900' and having eight digits.
- Short codes for general services beginning with '1' or '999' and having four digits.
- Short codes for text messages beginning with '8' and having five digits.

OMR 1=AED 9.54.

3.12.2 Fixed and mobile numbers

TRA allocates fixed numbers in blocks of 1,000 and mobile numbers in blocks of 100,000. In 2007/1428, TRA created three categories of special mobile numbers (called 'diamond', 'gold' and silver'). There is an allocation fee of:

- OMR 100 for a diamond number.
- OMR 75 for a gold number.
- OMR 25 for a silver number.

In addition there is an allocation fee of OMR 0.025 for each fixed or mobile number. The mobile service providers extract the special numbers and pay accordingly.

In 2013/1434, TRA issued a consultation document about replacing this system, and the fees for special numbers, with auctions. Whether to do this remains under review. In 2014/1435, TRA promulgated new numbering regulations that did not change matters in this respect.

Mobile numbers may be ported. However, as the consultation document, *Special Number Auctions*, explains, following a decision in 2011/1432 they must not be traded, except between husbands and wives, between persons and first degree cousins, between government organisations (or their employees) and companies in which the country is a shareholder (or their employees), and between first degree companies (under the classification of the Ministry of Transport and Communications) and their retirees or employees.

Table 25 shows the definitions of the special numbers by TRA. It is drawn from a verbal description in the TRA regulations. That description appears to omit obvious patterns (such as 'aabbbb**'), duplicate certain patterns (by including '**ababab' among both the diamond patterns and the gold patterns, for example), make some dubious categorisations (by regarding both 'abababab' and 'ababab**' as diamond patterns, for example) and contain ambiguities (by suggesting either '*aaa*bbb' or 'ab*ab*ab' as a gold pattern, for example). The patterns, constraints and capacities in the table might therefore not be exactly those intended by TRA. The following points from the ordering and categorisation might have general interest:

- Numbers that are uniform near their ends are higher in the ordering and categorisation than numbers that are uniform near their beginnings.
- Numbers that include repetitions or clusters are mainly higher in the ordering and categorisation than numbers that include sequences.
- Numbers that include ascending sequences are higher in the ordering and categorisation than numbers that include descending sequences.

Table 25 Special mobile numbers as defined by the regulator in Oman

Category	Pattern	Capacity
Diamond	***aaaaa	10,000
	**aaaaa*	9,000
	*aaaaa**	9,000
	aaaaa***	9,000
	*aa*aaaa	900
	aa**aaaa	810
	aa*aaaa*	810
	abababab	90
	**abbbba	9,000
	**aabbbb	9,000
	*aa*bbbb	7,200
	aa**bbbb	7,290
	aabbbb	7,200
	aa*bbbb*	6,480
	**ababab	8,910
	ababab	8,100
	ababab**	8,100
Gold	**a*aaaa	8,100
	**aaa*aa	9,000
	*a*aaaa*	6,480
	*aaa*aa*	9,000
	****aaaa	65,610
	***aaaa*	65,610
	*aaa*aaa	0
	aaa*aaa*	8,100
	abbbba	9,000
	**ababab	0
	aaababaa	90
	*aaa*bbb	8,100
	aaa*bbb*	7,290
	abbaabba	90
	aabaaaba	90
	abbcabbc	720
	acbcacbc	720
	abacabac	720
	aabbaabb	90

	aabcaabc	720
	abccabcc	720
	abcaabca	720
	abcdabcd	5,040
	abbabaab aababaaa	90 90
	abbcbcab acbcbcac	720 720
	abacacab	720
	aabbbbaa	0
	aabcbcaa	720
	abcacaab	720
	abccccab	720
	abcdcdab	5,040
	$**a_0a_1a_2a_3a_4a_5$ $*a_0a_1a_2a_3a_4a_5*$ $a_0a_1a_2a_3a_4a_5**$	500 423 460
Silver	**aaaa**	72,000
	***a*aaa ***aaa*a **a*aaa* **aaa*a*	89,100 90,000 80,190 80,190
	*****aaa ***aa*aa ****aaa* **aa*aa*	801,900 81,000 801,900 72,900
	**aabaab *aab*aab	0 8,100
	**abaaba *aba*aba	9,000 7,200
	**abbabb *abb*abb	0 8,100
	**abcabc *abc*abc	72,000 64,800
	***abbba **abbba*	0 19,900
	ab*ab*ab ab**abab abab**ab	8,010 8,820 8,730

	$**a_5a_4a_3a_2a_1a_0$ $*a_5a_4a_3a_2a_1a_0*$ $a_5a_4a_3a_2a_1a_0**$	500 423 460

Ooredoo lets customers select special mobile numbers from lists with prices equal to the allocation fees payable to TRA. In addition it lets customers select their fixed numbers from lists free of charge.

Omantel does not appear to have any service offerings that distinguish special numbers from other numbers.

There are some classified advertisements selling special mobile numbers. At the time of writing '95957971' was on offer for OMR 50.

3.12.3 Toll free and premium rate numbers

TRA allocates toll free and premium rate numbers individually. There is an allocation fee of OMR 25 for each toll free or premium rate number. There are no categories of special numbers.

3.12.4 Short codes

TRA allocates short codes individually. There is an allocation fee of OMR 250 for each short code. There are no categories of special numbers.

3.13 Pakistan

3.13.1 Background

The regulator is the Pakistan Telecommunication Authority (PTA). The main service providers are Mobilink, Telenor, Ufone (controlled by Etisalat), Warid and Zong.

In national dialling, fixed and mobile numbers are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '2', '4', '5', '6', '7' or '9' and having nine or ten digits.
- Mobile numbers beginning with '3' and having ten digits.
- Toll free numbers beginning with '800' and having eight digits.
- Premium rate numbers beginning with '900' and having eight digits.
- Short codes for general services beginning with '1' and having two, three or four digits.

There are also universal access numbers.

PKR 1=AED 0.0360.

3.13.2 Fixed and mobile numbers

PTA allocates fixed numbers in blocks of 10,000 or 100,000 and mobile numbers in blocks of 1,000,000. There are an application fee of PKR 1,000 for each application, and an annual fee of PKR 0.5 for each fixed or mobile number. There are no categories of special numbers.

Mobile numbers may be ported. They may also be traded; however, they are allocated to Subscriber Identifiy Modules (SIMs) for sale only in particular areas of the country.

Ufone lets customers enter between four and seven digits of their favoured mobile numbers to obtain lists from which they can select numbers without extra charge. The numbers available include, for example, '3312912129', '3312020939', '3312012349', '3313786543', '14786633', '3313786644', '3314786668' and '3310078765'. There are no numbers available including '3333', '331331' or '786786'.

Warid lets customers select special mobile numbers from lists with prices sampled in Table 26.

Table 26 Special mobile numbers as defined by one service provider in Pakistan

Category	Price	Pattern
Platinum	PKR 11,950	*****aaaaa ****aaaaa* ***aaaaa**
Gold	PKR 2,390	***aaaa*** *abbbbba** *a**abbbb* **aa**bbbb
Silver	PKR 597.50	a*a**aaa** ab*b*abbb* abbaccc*b* abba***aaa *a*abbb*** *a*a*bbb** *a*a**bbb* *a*a***bbb

Mobilink, Telenor and Zong do not appear to have any service offerings that distinguish special numbers from other numbers.

The number trader Nadeem lets customers select special mobile numbers from lists with prices sampled in Table 27.

Table 27 Special mobile numbers as defined by one number trader in Pakistan

Category	Price	Pattern
	PKR 3,500	ab*abb*786
	PKR 1,900	aa*a**bbba aabb**ccc* aa***bb007 $aab_0b_1b_2cccdd$
	PKR 1,300	aabbcc**** aa*b**b*cc aa*bc*c**b aa***bb*cc
	PKR 1,200	ab*a**bbba ab*a**b*cc
	PKR 1,000	aa***aaabb aa*a*bbb** aab**bbb** aabb*cc*c* aabbcc**dd
	PKR 900	$a_0a_1a_2a_0a_3a_4a_1a_3bb$ abbca*cacc abbca*caa* a*bab*c**c abbaccc***
	PKR 800	$a_3a_2a_1a_3a_0$**** aa*a****bb aa***bb*cc aa*abc*cbc aab*ac**bc
	PKR 700	abbacaacc* abbacc**dd
	PKR 600	$a_0a_1a_2a_0a_3a_4a_1a_4$** ab*a**bb** *a**babcc* *a*bcbcb*b *a*bcacbdd
	PKR 500	a*aabbb***
	PKR 400	ab**accb**
	PKR 300	ab*ab***a* abaab**c*c abba*aacc* abcab**caa abbacddeec
	PKR 200	abba*cc*** abaabc**bc

The Nadeem lists also include numbers incorporating '786' or '007'.

Number management practices

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 28.

Table 28 Special mobile numbers as priced in classified advertisements in Pakistan

Price	Number
PKR 35,000	3334444498
PKR 20,000	3452000009
PKR 15,000	3236666697
PKR 10,000	3364333321
PKR 6,000	3330485555
PKR 2,000	3350078007
PKR 1,300	3330222977
PKR 500	3330306030

3.13.3 Toll free and premium rate numbers

PTA allocates toll free numbers individually and premium rate numbers in blocks of 10. There are an application fee of PKR 1,000 for each application, and an annual fee of PKR 5,000 for each toll free or premium rate number. There are no categories of special numbers.

3.13.4 Short codes

PTA allocates short codes individually. There are an application fee of PKR 1,000 for each application, and an annual fee of PKR 5,000 for each short code. There are no categories of special numbers.

3.14 Qatar

3.14.1 Background

The regulator is the Communications Regulatory Authority (CRA), which succeeded ictQatar. The main service providers are Ooredoo and Vodafone.

The numbering plan provides:

- Fixed numbers beginning with ‘4’ and having eight digits.
- Mobile numbers beginning with ‘3’, ‘5’, ‘6’ or ‘7’ and having eight digits.
- Toll free numbers beginning with ‘800’ and having seven digits.
- Premium rate numbers beginning with ‘900’ and having seven digits.
- Short codes for general services beginning with ‘1’ and having three or four digits.

QAR 1=AED 1.01.

3.14.2 Fixed and mobile numbers

CRA allocates fixed numbers in blocks of 1,000 or 10,000 and mobile numbers in blocks of 100,000. There are no fees. There are no categories of special numbers.

Mobile numbers may be ported. Number transfers between service providers require approval by TRA.

In 2013/1434, ictQatar issued a consultation document about introducing fees for numbers. There would have been an application fee of QAR 2,000 for each application (or QAR 100,000 if the application was for a number block out of order), and an annual fee of QAR 1 for each fixed or mobile number. ictQatar ultimately decided not to introduce fees.

Ooredoo lets customers select special mobile numbers from lists with prices sampled in Table 29.

Table 29 Special mobile numbers as defined by one service provider in Qatar

Category	Price	Pattern
	QAR 5,000	aabcccbc
	QAR 1,500	aabbb*cc aa*bbbcc
	QAR 1,000	aaaa**** aa*aaa** aa**bbb*
	QAR 700	aabbbc*c *aabbcc* *aabb*cc
	QAR 500	aaab*bba aabbaa** a**bbba* abcbac** abcb**ac
	QAR 300	aaabb*** aa*bbb** aa***bbb aa**bbcc

Ooredoo also sells special mobile numbers in auctions for charity; in 2006/1427 '6666666' fetched QAR 10,000,000 and in 2012/1433 numbers beginning with '33' or '66' fetched between QAR 200,000 and QAR 1,400,000 each. In 2010/1431 the first digit of each fixed or mobile numbers was repeated, so '6666666' became '66666666' and all earlier fixed and mobile numbers became slightly more attractive.

Vodafone lets customers select special ('star') mobile numbers from lists sampled in Table 30.

Table 30 Special mobile numbers as defined by another service provider in Qatar

Category	Price	Pattern
Diamond	QAR 1,500	aaa***** aaba**ba aa*abbb* aa*bb*a a*bbb*a* *abbb*a*
Platinum	QAR 1,000	aaba**ba a*b*abaa abc*bcba abbcddac abccbab* *abcacbc
Gold	QAR 500	aab**ab* aa*bb*a* abcbac** abcb**ac
Silver	QAR 300	aaba**b* aab*a**b aabb***a aa*b*ba*

Vodafone also sells special mobile numbers in auctions for charity; in 2009/1430 forty numbers raised a total of QAR 11,586,000 and subsequently numbers for auction were valued between QAR 2,000 and QAR 50,000 each.

There are some classified advertisements selling special mobile numbers. At the time of writing '77470666' was on offer with no reserve price.

3.14.3 Toll free and premium rate numbers

CRA allocates toll free and premium rate numbers individually. There are no fees. There are no categories of special numbers.

Under the proposal in the consultation document about introducing fees for numbers there would have been an application fee of QAR 2,000 for each application (or QAR 100,000 if the application was for a number out of order), and an annual fee of QAR 10 for each toll free or premium rate number. ictQatar ultimately decided not to introduce fees.

3.14.4 Short codes

CRA allocates short codes individually. There are no fees. There are no categories of special numbers.

Under the proposal in the consultation document about introducing fees for numbers there would have been an application fee of QAR 2,000 for each application (or QAR 100,000 if the application was for a number out of order), and an annual fee of:

- QAR 100,000 for a three-digit short code.
- QAR 10,000 for a four-digit short code.
- QAR 1,000 for a five-digit short code.

Thus the fee would have been multiplied by ten for each reduction in number length by one digit. ictQatar ultimately decided not to introduce fees.

3.15 Saudi Arabia

3.15.1 Background

The regulator is the Communication and Information Technology Commission (CITC). The main service providers are Saudi Telecom, Mobily (controlled by Etisalat) and Zain.

In national dialling, fixed and mobile numbers are prefixed by '0'. The numbering plan of 2011/1432 provides:

- Fixed numbers beginning with '1' and having nine digits.
- Mobile numbers beginning with '5' or '6' (but not '52' or '62') and having nine digits.
- Toll free numbers beginning with '800' and having ten digits.
- Premium rate numbers beginning with '700' and having eight digits.
- Short codes for general services beginning with '1' or '9' (but not '92') and having three, four, five or six digits.
- Short codes for text messages beginning with '5', '6', '7' or '8' and having four or six digits.

There are also shared cost, shared revenue, nomadic, machine-to-machine, personal and public mobile data numbers.

SAR 1=AED 0.98.

3.15.2 Fixed and mobile numbers

CITC allocates fixed numbers in blocks of 1,000 or 10,000 and mobile numbers in blocks of 100,000. There are an allocation fee of SAR 0.10, and an annual fee of SAR 0.30, for each fixed or mobile number.

Fixed and mobile numbers may be ported. However, they must not be traded; the CITC regulations, *National Numbering Plan 2011/1432*, allow service providers to remove numbers that have been transferred between customers.

In 2003/1424, CITC published a national numbering plan that identified special ('golden') fixed and mobile numbers. In 2011/1432, CITC changed the national numbering plan to one that no longer did this. There is no special fee for a special number.

Table 31 shows the definitions of the special fixed and mobile numbers by CITC. It assumes that fixed numbers have nine digits each; when the numbers were originally identified, fixed numbers had only eight digits each, but the special number for them were essentially some of those in the table except for losing a digit. The following points from the ordering might have general interest:

- Numbers that are uniform near their ends are lower in the ordering than numbers that are uniform near their beginnings.
- Numbers that include repetitions or clusters of digits are mainly higher in the ordering than numbers that include sequences.
- Numbers that include ascending sequences are no higher in the ordering than numbers that include descending sequences.

Table 31 Special fixed and mobile numbers as defined by the regulator in Saudi Arabia

Category	Pattern	Capacity
	aaaaaaaaa	10
	aaaaaaaa*	90
	aaaaaaabb	90
	aaaaaabbb	90
	*aaaaaaaa	90
	**aaaaaaa	900
	***aaaaaa	9,000
	****aaaaa	90,000
	*****aaaa	900,000
	aaaaaaa	810
	*aaaaaa**	8,100
	**aaaaaa*	8,100
	aaaaa	81,000
	***aaaaa*	81,000
	***aaaa**	810,000
	aaaa*	810,000
	****aaaa*	810,000
	*abababab	900
	***ababab	89,100
	*****abab	8,910,000
	aabaabaab	90

	abaabaaba	90
	abbabbabb	90
	abcabcabc	720
	***aaabbb	90,000
	*$a_0a_1a_2a_3a_4a_5a_6a_7$ *$a_7a_6a_5a_4a_3a_2a_1a_0$	30 30
	**$a_0a_1a_2a_3a_4a_5a_6$ **$a_6a_5a_4a_3a_2a_1a_0$	370 370
	***$a_0a_1a_2a_3a_4a_5$ ***$a_5a_4a_3a_2a_1a_0$	4,600 4,600
	****$a_0a_1a_2a_3a_4$ ****$a_4a_3a_2a_1a_0$	55,000 55,000
	*****$a_0a_1a_2a_3$ *****$a_3a_2a_1a_0$	640,000 640,000

Saudi Telecom lets customers select special fixed numbers from lists with prices sampled in Table 32.

Table 32 Special fixed numbers as defined by one service provider in Saudi Arabia

Location	Price	Pattern
Jeddah or Riyadh, for example	SAR 990	***aabbcc
Makkah or Medina, for example	SAR 290	******aaa *****abab *****$a_0a_1a_2a_3$

Saudi Telecom lets customers select special mobile numbers from lists with prices sampled in Table 33.

Table 33 Special mobile numbers as defined by one service provider in Saudi Arabia

Category	Price	Pattern
Gold	SAR 9500	*aabbccdd
Bronze	SAR 2500	*****aaaa
Special	SAR 600	*aaaa**** **aaabbcc
Economy	SAR 290	*****abab

Saudi Telecom also sells special mobile numbers in auctions; recently one fetched SAR 300,000 in an auction of sixty-four numbers in three categories (called 'diamond', 'gold' and 'silver') that raised SAR 5,000,000 and had a minimum reserve price of SAR 3,000.

Etisalat and Zain include special numbers in particular tariff packages.

There are some classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 34.

Table 34 Special mobile numbers as priced in classified advertisements in Saudi Arabia

Price	Number
SAR 150,000	555555256
SAR 1,000	582845555
SAR 700	570077575
SAR 70	799786568

3.15.3 Toll free and premium rate numbers

CITC allocates toll free and premium rate numbers individually. There are an allocation fee of SAR 0.10, and an annual fee of SAR 0.30, for each toll free number, and an allocation fee of SAR 1, and an annual fee of SAR 3, for each premium rate number.

3.15.4 Short codes

CITC allocates short codes individually. There is an allocation fee of:

- SAR 300,000 (changed from SAR 100,000) for a three-digit short code.
- SAR 150,000 (changed from SAR 1,000) for a four-digit short code.
- SAR 50,000 (changed from SAR 100) for a five-digit short code.
- SAR 5,000 (changed from SAR 10) for a six-digit short code.

There is also an annual fee of:

- SAR 150,000 (changed from SAR 300,000) for a three-digit short code.
- SAR 100,000 (changed from SAR 3,000) for a four-digit short code.
- SAR 15,000 (changed from SAR 300) for a five-digit short code.
- SAR 1,500 (changed from SAR 30) for a six-digit short code.

Thus the fee is multiplied by ten for each reduction in number length by one digit, except in the case of three-digit and four-digit short codes.

In 2011/1432, CITC changed the national numbering plan to one that no longer identified special fixed and mobile numbers but now instead identified special ('vanity') short codes. There is no special fee for a special short code; however, CITC can choose to limit how many such codes are allocated to any one service provider.

Table 35 shows the definitions of the special short codes by CITC. They involve the use of particular digits ('0' and '1'), repetitions, and sequences.

Table 35 Special short codes as defined by the regulator in Saudi Arabia

Category	Pattern	Capacity
	160a	10
	170a	10
	161a	10
	171a	10
	16aa with a≠0 and a≠1 and a≠6	7
	17aa with a≠0 and a≠1 and a≠7	7
	16a0 with a≠0 and a≠1 and a≠6	7
	17a0 with a≠0 and a≠1 and a≠7	7
	16a6 with a≠0 and a≠1 and a≠6	7
	17a7 with a≠0 and a≠1 and a≠7	7
	1678	1
	1789	1
	1771a	10
	1777a	10
	166a0 with a≠0 and a≠1 and a≠6	7
	177a0 with a≠0 and a≠1 and a≠7	7
	166a6 with a≠0 and a≠1 and a≠6	7
	177a7 with a≠0 and a≠1 and a≠7	7
	166aa with a≠0 and a≠1 and a≠6	7
	177aa with a≠0 and a≠1 and a≠7	7
	16678	1
	17789	1
	16600a	10
	17700a	10
	16606a	10
	17707a	10
	1660a0 with a≠0 and a≠1 and a≠6	8
	1770a0 with a≠0 and a≠1 and a≠7	8
	1660aa with a≠0 and a≠1 and a≠6	8
	1770aa with a≠0 and a≠7	8
	166016	1
	177017	1

3.16 Singapore

3.16.1 Background

The regulator is the Infocomm Development Authority (IDA). The main service providers are M1, SingTel and StarHub.

The numbering plan provides:

- Fixed numbers beginning with '3' and having eight digits.
- Mobile numbers beginning with '8' or '9' and having eight digits.
- Toll free numbers beginning with '1800' and having eleven digits.
- Premium rate numbers beginning with '1900' and having eleven digits.
- Short codes for general services beginning with '0' or '1' and having three, four or five digits.
- Short codes for emergency services beginning with '99' and having three digits.

There are also nomadic, machine-to-machine and international toll free numbers.

SGD 1=AED 2.70.

3.16.2 Fixed and mobile numbers

IDA allocates fixed and mobile numbers in blocks of 10,000. IDA regards each block as containing 468 special numbers. Each of these numbers has a notional price of SGD 50, so each block has a minimum price of SGD 24,300. Numbers are allocated administratively (in response to an application) or sold in auctions. There is an allocation fee of SGD 24,300 for a block that is allocated administratively. In an auction, IDA releases about 100 blocks for auction at once, according to its schedule. Each block released by IDA has a reserve price of SGD 25,000 (which results from rounding up SGD 24,300 to simplify bidding). In addition, service providers can trigger auctions by requesting blocks that IDA has not released; such a block has a reserve price of SGD 150,000 unless its numbers begin with 'aaaa' (when the reserve price is SGD 500,000).

Fixed and mobile numbers may be ported. They may also be traded; the IDA regulations prohibit service providers from withdrawing or changing the numbers of customers except under clearly defined conditions (which trading is not among).

Table 36 shows the definitions of the special fixed and mobile numbers by IDA. It is drawn from the regulations. The same patterns specify the special numbers for each block.

Table 36 Special fixed and mobile numbers as defined by the regulator in Singapore

Category	Pattern	Capacity
	****aaaa	100,000
	*****aaa	900,000
	****aaa*	900,000
	****aabb	900,000
	****abab	900,000
	****abba	900,000
	****8a88 with a≠5 and a≠8	80,000
	****88a8 with a≠5 and a≠8	80,000
	****1234	10,000
	****1628	10,000
	****1168 ****1668	10,000 10,000
	****1288 ****1388 ****1688 ****1788 ****1988 ****3288	10,000 10,000 10,000 10,000 10,000 10,000

StarHub lets customers select special fixed numbers represented by ****8888 (called 'gold') or *****88 (called 'silver') from lists with prices such as:

- SGD 415 for a gold number.
- SGD 95 for a silver number.

StarHub also sells special mobile numbers by auctions. It releases five tier 1 numbers and fifty tier 2 numbers for auction each month; typically those released for auction have '8' as the first digit and have a repetition in the first four digits. Only 5% or 10% of them might be sold in the auction. There are reserve prices of:

- SGD 888 for a tier 1 number released for auction.
- SGD 488 for a tier 2 number released for auction.

Table 37 shows the definitions of the special numbers by StarHub. It is drawn from a description on the StarHub web site.

Table 37 Special mobile numbers as defined by one service provider in Singapore

Category	Pattern	Capacity
Tier 1	****aaaa	100,000
Tier 2	*****aaa	900,000
	****aaa*	900,000
	****aabb	900,000

	****abab	900,000
	****abba	900,000
	****8*88	90,000

M1 lets customers select special mobile numbers from lists with prices sampled for Table 38.

Table 38 Special mobile numbers as defined by another service provider in Singapore

Category	Price	Pattern
	SGD 201 – SGD 500	a***aaa* a*aabb*a a*a**bbb *a*a*bbb
	SGD 151 – SGD 200	a****aaa *a**aaa* *a***aaa a*b*aaab ****aabb ****abab
	SGD 101 – SGD 150	****aa*a a*aabb** *aaabb*a **aa*bb* ****abba ababcc**
	SGD 51 – SGD 100	a*aab*b* *aaab*b* **aabb** **aa*bb* abaac*cb abccaa*b
	SGD 30 – SGD 50	abbab*a* abba*b** abba**b* abba*c*c a*bcabc*

M1 also sells special mobile numbers by auctions with reserve prices, sampled for Table 39.

Table 39 Special mobile numbers as defined by another service provider in Singapore

Category	Price	Pattern
	SGD 5,000	abaabaaa
		ababaabb
		ababbbba
		abbbabab
		abbbbaba
		abbbbbaa
	SGD 3,000	*aaaaaa*
		abbabbaa
		abbabbbb
		abba*aaa
		*a*abbbb
	SGD 1,000	a**aaaa*
		a*aabbbb
		a*abbbbb
		a*bbaaa*
		*aa*bbbb
		**aabbbb
	SGD 800	a*aaaaa*
		*a**aaaa
		**a*aaaa
		a*aaabab
		abbba*aa
		*aa*bbbb
		abbbacac
		$a_7a_6a_5a_4a_3a_2a_1a_0$

Singtel does not appear to have any service offerings that distinguish special numbers from other numbers, though it seems to have done so in the past.

There are many classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 40.

Table 40 Special mobile numbers as priced in classified advertisements in Singapore

Price	Number
SGD 18,888	97800000
SGD 8,888	87350555
SGD 3,888	91555050
SGD 988	81818385
SGD 900	90090119
SGD 388	98999696
SGD 188	83797926
SGD 99	83877000

3.16.3 Toll free and premium rate numbers

IDA allocates toll free numbers individually and premium rate numbers in blocks of 10,000, except that numbers ending with 'aaaaaaa' are sold in auctions with a reserve price of SGD 20,000 for each of them.

3.16.4 Short codes

IDA allocates short codes individually. Some are allocated in lotteries; others are allocated in auctions with a reserve price of SGD 50,000 for each of them.

3.17 United Kingdom

3.17.1 Background

The regulator is the Office of Communications (Ofcom), which succeeded the Office of Telecommunications (Oftel). The main service providers are British Telecommunications, Everything Everywhere, O2, Vodafone and 3 (controlled by Hutchison).

In national dialling, fixed and mobile numbers, and toll free and premium rate numbers, are prefixed by '0'. The numbering plan provides:

- Fixed numbers beginning with '1' or '2' and having ten digits.
- Mobile numbers beginning with '7' (but not '70' or '76') and having ten digits.
- Toll free numbers beginning with '80' and having nine or ten digits.
- Premium rate numbers beginning with '9' and having ten digits.
- Short codes for general services beginning with '1' and having three, four, five or six digits.
- '999' for emergency services.

There are also geographic rate, shared cost, shared revenue, nomadic, corporate, personal and paging numbers. Short codes for text messages are outside the numbering plan.

GBP 1=AED 5.67.

3.17.2 Fixed and mobile numbers

Ofcom allocates fixed numbers in blocks of 100, 1,000 or 10,000 and mobile numbers in blocks of 100,000. For experimental purposes there is an annual fee of GBP 0.10 for a fixed number in a range that is in very short supply (and that is therefore now allocated in blocks of 100); otherwise there are no fees. There are no categories of special numbers.

Fixed and mobile numbers may be ported. They may also be traded; the Ofcom regulations prohibit service providers from changing the numbers of customers except under clearly defined conditions such as breach of contract.

In 1999 Oftel set up a working group to consider charging for individual numbers. The proposals discussed would have created three categories of special numbers (called 'platinum', 'gold' and silver'). For instance:

- A number represented by '*******999' would have been in the platinum category.
- A number represented by '****aaaaaa', '***aaa*aaa', '****aaabbb', '***abbaaaa', '****aabbcc' or '***abbaccc' would have been in the gold category.
- A number represented by '***abbaacc' or '***abbcaca' would have been in the silver category.

There would also have been categories of numbers that could be mapped to words using the standard keypad and charges for selecting any number specially.

The proposals were intended for toll free numbers but could be applied also to fixed and mobile numbers. For fixed and mobile numbers the proposals suggested an annual fee of GBP 1,000 for each block of 10,000 numbers (based on the long-run incremental cost of extending the number supply) and observed that some service providers were charging annual fees for special numbers that could be as high as GBP 1,000 for such a number. A related unpublished economic study noted the difficulty in determining fees for special numbers administratively and favoured auctions.

The proposals were never formulated fully and ultimately lapsed because Oftel was not certain that it had legal powers to charge for numbers. Though these powers were subsequently granted to Ofcom, the proposals were never revived. The only numbers for which there are fees are fixed numbers for areas where the supply of numbers is almost exhausted: as an experiment to investigate the effect on demand there is an annual fee of GBP 0.1 for each such number.

Companies such as Cherished Mobile Numbers, Elite Numbers, TTNC, UK Mobile Numbers and VIP Numbers let customers select special mobile numbers from lists (and TTNC provides lists of special fixed numbers, too). They ascribe very different meanings to terms such as 'platinum', 'gold' and 'silver'. They typically have prices between GBP 30 and GBP 1,500. The lists from VIP Numbers, sampled for Table 41, are more extensive and well described.

Table 41 Special mobile numbers as defined by one number trader in the United Kingdom

Category	Price	Pattern
	GBP 3,500	**aaaaaaa*
	GBP 2,500	****aaaaaa **aaaabbbb *aaabbbccc
	GBP 2,000	aaa*aaaaa* *abcabcabc
	GBP 1,500	***$a_0a_1a_2a_3a_4a_5a_6$ ***$a_6a_5a_4a_3a_2a_1a_0$ *abbbabbb* **abababba *a$b_0b_1b_2b_3b_4b_5b_6$a
	GBP 500	**$a_0a_1a_2a_3a_0a_1a_2a_3$ aababbbbba abbaabaaab abbabbcccc abbcabcabc
	GBP 400	*$a_3a_3a_2a_2a_1a_1a_0a_0$ ****$a_5a_4a_3a_2a_1a_0$ *aabbbbccc abbaabaaab abbabbcccc abbcabcabc
	GBP 350	**$a_0a_1a_2a_3a_4a_5$** ****$a_0a_1a_2a_0a_1a_2$ **aaaabbba aabccbbcc* abc*abcabc *ab$c_2$ab$c_1$abc_2 *aaabbccddd
	GBP 350 – GBP 500	*****00000 **a0a0a00a ****a0a0a0
	GBP 300	*a*baaaaab *a*bababab ***abababa **aabcbcbc *aabcccbbb.

	GBP 250	**$a_6a_5a_4a_3a_2a_1a_0$* a*aabbbbaa aaa$b_0b_1b_2b_3b_4b_5$* *a$b_0$aa$b_1$aab_2a **ab_0ab_0ab_0ab_1 ***aaaaabb ***aabbbbb a*bbbccaaaa aaabb$c_0c_1c_2c_3c_4$ *abcacbacb abcdeabcde
	GBP 200	***aaaaa*a* ***aaaa*aa ***a*aaaaa *a*baabaab aabaaabccc a*babacccc *abacabacc **aabbccdd
	GBP 200 – GBP 300	****aaabbb ****ababab
	GBP 175	aabbcccc*c ab*abcabca *aabcabca* **abcaabca
	GBP 150	****$a_0a_1a_2a_0a_1a_2$ ****$a_0a_1a_2a_2a_1a_0$ ****$a_2a_1a_0a_0a_1a_2$ aaa***bbbb a*babab aaa **aabbbbb* ***abbbbba ****aabaab ****abaaba ****abbabb *aa*bbcccc ****abcabc **aabcdbcd

	GBP 125	****$a_2a_1a_0a_2a_1a_0$ ****$a_0a_0a_1a_1a_2a_2$ aabbcccc** a*bacacaba a**abbbbcc **abababcc aabbccdd** aabbcdcd*d
	GBP 85	**$ab_0ab_0ab_1ab_2$ **$ab_1ab_2ab_1ab_0$ **$ab_2ab_2ab_1ab_0$ **abcdabcd
	GBP 50	abbabba**a **ababab** abcabca**a *abcaaabca **$ab_2ab_2ab_1ab_0$ abcaaacba* **abcdabcd
	GBP 30	aaa***abbb aaa**b*bbb aa***aabbb
	GBP 20	aaa****bbb

The VIP Numbers lists also include numbers incorporating '168', '786', '147', '911', '007', '365' or '247'. As the enumeration above indicates, '0' sometimes generates higher prices, and two very similar numbers sometimes generate very different prices. To some extent the lists also suggest that:

- Numbers that are uniform near their ends are higher in price than numbers that are uniform near their beginnings.
- Numbers that include repetitions or clusters are mainly higher in price than numbers that include sequences.
- Numbers that include ascending sequences are higher in price than numbers that includen descending sequences.

All this might result from careful calculation and long experience, but it might also happen if numbers are put in the lists on different occasions or by different people.

There are many classified advertisements selling special mobile numbers. Typical prices at the time of writing are exhibited in Table 42.

Table 42 Special mobile numbers as priced in classified advertisements in the United Kingdom

Price	Number
GBP 3,000	786*786786
GBP 500	7740000006
GBP 400	7770070*00
GBP 100	744432444*
GBP 60	7774477*11
GBP 50	77767766*6
GBP 30	7***337777
GBP 20	7919193454

3.17.3 Toll free and premium rate numbers

Ofcom allocates toll free and premium rate numbers in blocks of 10,000. There are no fees. There are no categories of special numbers.

Toll free and premium rate numbers may be ported. They may also be traded; the Ofcom regulations prohibit service providers from changing the numbers of customers except under clearly defined conditions such as breach of contract.

3.17.4 Short codes

Ofcom allocates short codes individually. There are no fees. There are no categories of special numbers.

4 Formulating patterns

This section presents the theory that underpins the investigations in Section 5 and Section 6. It applies that theory in order to classify and band patterns in the complete eight-digit numbering space, by calculating their capacities as well as the capacities of smaller numbering spaces having numbers with four, five, six or seven digits. In doing so it demonstrates a distinctive new basis for the analysis of special numbers. A slight familiarity with mathematical ideas would help with understanding it.

4.1 Capacity calculation

4.1.1 Patterns as sets

Descriptions of numbers can use the notation developed earlier in this report. However, that notation is verbose and unsuitable for analysing the complex patterns needed in a systematic treatment. The notation developed below provides a much more succinct alternative, with rules for calculating capacities.

A pattern represents numbers, and the numbers form a set. The pattern is therefore, in effect, a set of numbers. Accordingly patterns can be subjected to the usual operations on sets, such as union, intersection, and difference. In fact these operations already underlie the tables in this report: a combination of constraints using 'or' defines the union of two sets of numbers, a combination of constraints using 'and' defines the intersection of two sets of numbers and an implicit constraint excluding the numbers represented by patterns in earlier rows defines the difference between two sets of numbers (which are those "apparently" represented by the pattern in the current row and those represented by the patterns in the earlier rows).

For this, more notation is useful. Here upper-case letters ('P', 'Q', 'R' and so on) signify patterns, such as '2110' and 'abcd' (though later in this report they have other roles). Then 'P|Q' (pronounced "P or Q") signifies the union of patterns 'P' and 'Q': it represents the numbers that are represented by either 'P' or 'Q'. Also 'P&Q' (pronounced "P and Q") signifies the intersection of patterns 'P' and 'Q': it represents the numbers that are represented by both 'P' and 'Q'. Finally 'P\Q' (pronounced "P except Q") signifies the difference of patterns 'P' and 'Q': it represents the numbers that are represented by 'P' but not 'Q'. For instance:

- If 'P' signifies the pattern '2110' and 'Q' signifies the pattern '2220' then 'P|Q' signifies the union of the patterns '2110' and '2220'.
- If 'R' signifies the union of the patterns '2110' and '2220' and S signifies the union of the patterns '2220' and '2230' then 'R&S' signifies the pattern '2120'.

- If 'R' signifies the union of the patterns '2110' and '2220' and S signifies the union of the patterns '2220' and '2230' then 'R\S' signifies the pattern '2110' and 'S\R' signifies the pattern '2230'.

As these examples illustrate, if 'P' and 'Q' are patterns then 'P' is the union of the patterns 'P&Q' and 'P\Q', which have no overlaps (so no number is represented by both 'P&Q' and 'P\Q'). In other words

P=(P&Q)|(P\Q).

Here '(' and ')' simply help to distinguish the parts of the formula.

4.1.2 Jumbles of patterns

Patterns can be joined together to form longer patterns; they are then said to be 'concatenated'. Each of the numbers represented by joining two patterns together is obtained by putting a number represented by the first pattern at the beginning of a number represented by the second pattern. For instance:

- If 'P' signifies the pattern '2110' and 'Q' signifies the pattern 'abcd' then 'PQ' signifies the pattern '2110abcd' and 'QP' signifies the pattern 'abcd2110'.
- If 'R' signifies the union of the patterns '2110abcd' and 'abcd2110' and 'S' signifies the union of the patterns '34' and 'ef' then 'RS' signifies the union of the patterns '2110abcd34', 'abcd211034', '2110abcdef', 'abcd2110ef' and 'SR' signifies the union of the patterns '342110abcd', '34abcd2110', 'ef2110abcd' and 'efabcd2110'; in other words, if 'R' signifies '2110abcd|abcd2110' and 'S' signifies '34|ef' then 'RS' signifies '2110abcd34|abcd211034|2110abcdef|abcd2110ef' and 'SR' signifies '342110abcd|34abcd2110|ef2110abcd|efabcd2110'.

Patterns can be joined together in any order. If 'P' and 'Q' are patterns, the 'jumble' of 'P' and 'Q', which is written as '[P§Q]', is the pattern that results from joining together 'P' and 'Q' in either order. Hence '[P§Q]' is defined by

[P§Q]=PQ|QP.

For instance:

- If 'P' signifies the pattern '2110' and 'Q' signifies the pattern 'abcd' then '[P§Q]' signifies the union of the patterns '2110abcd' and 'abcd2110'.
- If 'R' signifies the union of the patterns '2110abcd' and 'abcd2110' and 'S' signifies the union of the patterns '34' and 'ef' then '[R§S]' signifies the union of the patterns '2110abcd34', 'abcd211034', '2110abcdef', 'abcd2110ef', '342110abcd', '34abcd2110', 'ef2110abcd' and 'efabcd2110'; in other words, in other words, if 'R' signifies '2110abcd|abcd2110' and 'S' signifies '34|ef' then '[R§S]' signifies '2110abcd34|abcd211034|2110 abcdef|abcd2110ef|342110abcd|34abcd2110|ef2110abcd|efabcd2110'.

More generally, if 'P', 'Q', 'R', …, 'X', 'Y', 'Z' are patterns, their jumble, which is written as '[P§Q§R§…§X§Y§Z]', is a union of jumbles; each of those jumbles is obtained by joining together one of the patterns with the jumble of all of the remaining patterns. Hence '[P§Q§R§…§X§Y§Z]' is defined by:

[P§Q§R§…§X§Y§Z]=

P[Q§R§…§X§Y§Z] | Q[R§…§X§Y§Z§P] | … |

Y[Z§P§Q§R§…§X] | Z[P§Q§R§…§X§Y].

For instance:

- '[a§a]' is 'aa' (after removing duplicate patterns).
- '[a§b]' is the union of the patterns 'ab' and 'ba'.
- '[a§a§b]' is the union of the patterns 'aab', 'aba' and 'baa' (after removing duplicate patterns).
- '[a§a§a§b]' is the union of the patterns 'aaab', 'aaba', 'aaba' and 'baaa' (after removing duplicate patterns).
- '[a§a§b§b]' is the union of the patterns 'aabb', 'abba', 'abab', 'bbaa', 'baab' and 'baba' (after removing duplicate patterns).
- '[a§a§b§c]' is the union of the patterns 'aabc', 'aacb', 'abac', 'acab', 'abca', 'acba', 'bcaa', 'baac', 'baca', 'cbaa', 'caab' and 'caba' (after removing duplicate patterns).

Table 43 has the same content as Table 1 but uses jumbles for succinctness.

Table 43 The four-digit patterns represented by letters with '§'

Pattern	Capacity	Proportion (%)
aaaa	10	0.1
[a§a§a§b]	360	3.6
[a§a§b§b]	270	2.7
[a§a§b§c]	4,320	43.2
abcd	5,040	50.4

For all 'P',

[P§P]=PP.

For all 'P', 'Q' and 'R',

[P§Q§R]=[R§Q§P]=[P§R§Q].

However, '[[P§Q]§R]', '[P§[Q§R]]' and '[P§Q§R]' might represent different sets of numbers, because:

[[P§Q]§R]=PQR|QPR|RPQ|RQP.

[P§[Q§R]]=PQR|PRQ|QRP|RQP.

[P§Q§R]=PQR|PRQ|QRP|QPR|RPQ|RQP.

For instance:

- '[a§a]' is 'aa', so '[[a§a]§b]' is the union of the patterns 'aab' and 'baa'; as '[a§a§b]' is the union of the patterns 'aab', 'aba' and 'baa', '[[a§a]§b]' and '[a§a§b]' might represent different sets of numbers.
- '[a§b]' is the union of the patterns 'ab' and 'ba', so '[a§[a§b]]' is the union of the patterns 'aab', 'aba' and 'baa'; as '[a§a§b]' is the union of the patterns 'aab', 'aba' and 'baa', '[a§[a§b]]' and '[a§a§b]' represent the same set of numbers.

Thus '[' and ']' are perhaps more important than (' and ')' in distinguishing parts of formulas.

4.1.3 Factors in capacities

A jumble of patterns can be expanded into a union of patterns without jumbles (so '[aa§bb]', for example, can be expanded into 'aabb|bbaa'). All of the patterns in the union contain the same number of occurrences of each letter. In them the occurrences of letters are in different orders and might appear to represent different sets of numbers, but in fact the sets of numbers might overlap. For instance:

- 'aaab', 'aaba', 'abaa' and 'baaa' represent different sets of numbers (under the convention that different letters without subscripts represent different digits): there is no number represented by more than one of 'aaab', 'aaba', 'abaa' and 'baaa'.
- 'aabb' and 'bbaa' might appear to represent different sets of numbers, but if the constraints on 'a' and 'b' are the same then 'a' and 'b' represent the same set of digits and 'aabb' and 'bbaa' represent the same set of numbers; in particular, if there is no constraint on 'a' and 'b' other than the convention that different letters without subscripts represent different digits (which has the same effect on 'a' and 'b') then 'aabb' and 'bbaa' represent the same set of numbers.

A jumble of patterns, then, can be expanded into a union of patterns that contain the same number of occurrences of each letter and that represent possibly overlapping sets of numbers. The capacity of the jumble is the sum of the capacities of the patterns in the union, after taking into account the overlaps between the sets of numbers.

The capacity of the jumble can be calculated by finding the capacities and degrees of overlap of the patterns in the union. Usually this does not need to be done by expanding the jumble into the union; it can instead be done by multiplying together the following factors:

- The 'digit choice count', which is how many distinct combinations of digits can be represented by the (unordered and unrepeated) letters in the jumble.
- The 'letter order count', which is how many distinct orderings of letters can be obtained according to the degree of repetition of the letters in the jumble.
- The 'overlap extent', which is how much overlap there is between the sets of numbers represented by the jumble, bearing in mind the constraints on the digits represented by the letters.

The digit choice counts are simple to calculate; often they consist of factorials (where the constraints demand only that different letters without subscripts represent different digits) or powers (where there is no convention that different letters without subscripts represent different digits); factorials are written with '!' and powers are written with '^'. The letter order counts are also simple: they consist of factorials stating how often the occurrences of the letters in the pattern can be ordered. The overlap extents can be more complicated; they depend on how often different letters without subscripts occurring in the patterns have the same constraints.

Table 44 makes the calculation of digit choice counts, letter order counts and overlap extents apparent for the capacities in Table 43. In it:

- In the digit choice counts:
  - The factorial of ten ('10!') appears because letters can represent any of ten digits.
  - The factorial of nine, eight, seven or six is needed according to whether there are one, two, three or four distinct letters in the pattern.
- In the letter order counts:
  - The factorial of four ('4!') appears (before the '/') because the pattern consists of four occurrences of letters.
  - The factorial of four, three, two or one appears (after the '/') according to whether for each of the distinct letters in the pattern there are four, three, two or one occurrences of each of the letters in the pattern.

- In the overlap extents:
  - The factorial of two or four appears (after the '/') according to whether there are two or four distinct letters in the pattern that have the same constraints and occur the same number of times in analogous positions in the pattern.

The final point in this list, relating to two or four distinct letters, is a statement of a rule that requires care. To calculate overlap extents in general there can be a need to break up constraints and extract particular patterns. For instance:

- Both '[a§a§b§b]' and '[aa§bb]' have 'a' and 'b' in analogous positions, but '[aa§b§b]' does not and reduces to 'aabb|baab' (without 'bbaa') if 'a' and 'b' have the same constraints.

- Both '[a§a§b§b§c§c]' and '[abc§abc]' have 'a', 'b' and 'c' in analogous positions, but '[ab§ab§c§c]' does not and reduces to 'ababcc|abcabc|cababc| abccab|ccabab' (without 'cabcab') if 'a', 'b' and 'c' have the same constraints.

Table 44 The separation of digit choice counts, letter order counts and overlap extents

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
aaaa	10!/9!	4!/4!	1	10
[a§a§a§b]	10!/8!	4!/(3!×1!)	1	360
[a§a§b§b]	10!/8!	4!/(2!^2)	1/2!	270
[a§a§b§c]	10!/7!	4!/(2!×1!^2)	1/2!	4,320
abcd	10!/6!	4!/(1!^4)	1/4!	5,040

Separating digit choice counts, letter order counts and overlap extents lets each of them be changed with fewer repercussions: provided that the constraints change in the same way for distinct letters that occur the same number of times, the letter order count and overlap extent can be left unchanged while the digit choice count is changed. For instance:

- For the pattern '[a§a§a§b]' in Table 44, adding the constraint 'a=2' would change the digit choice count from '10!/8!' (or '10×9') to '9' without changing the letter order count and overlap extent: 'b' could still represent any digit other than '2' .

- For the pattern '[a§a§b§c]' in Table 44, adding the constraint 'b=0 and c=5 or b=5 and c=0' would change the digit choice count from '10!/7!' (or '10×9×8') to '8' without changing the letter order count and overlap extent; 'a' could still represent any digit other than '0' or '5'. The constraint could also be written as ‘bc∈[0§5]’, where ‘∈’ signifies ‘is a member of the set’.

- For the pattern ‘abcd’ in Table 44, adding a constraint that forced ‘a’, ‘b’, ‘c’ and ‘d’ to represent a permutation of consecutive digits would change the digit choice count from ‘10!/6!’ to ‘7’ (because there are seven sequences of four consecutive digits lying between ‘0’ and ‘9’) without changing the letter order count and overlap extent. The constraint could perhaps be written as ‘abcd∈[i_0§i_1§i_2§i_3]’ when ‘i_0’, ‘i_1’, ‘i_2’ and ‘i_3’ represent four consecutive digits.

When constraints do not change in the same way for distinct letters that occur the same number of times, both the digit choice count and the overlap extent are likely to need to be changed. The letter order count needs to change if the structure of the pattern changes; for instance ‘[a§a§b§b]’ provides six distinct combinations of ‘a’ and ‘b’ while ‘[aa§b§b]’ provides three.

4.1.4 Capacities for basic patterns

Table 45 provides patterns that represent the 10,000 four-digit numbers in such a way that each number is represented by one and only one pattern. The systematic technique for doing this considers the different quantities of each letter that can make up a pattern mentioning letters only. In particular:

- Two such patterns that mention different quantities of the same letter (‘aaab’ and ‘aabb’, for example) represent disjoint sets of digits under the convention that different letters without subscripts represent different digits.

- Two such patterns that mention the same quantity of different letters in the corresponding order but that are otherwise identical (‘aabb’ and ‘bbaa’, for example) represent the same set of digits if the letters are subject to the same constraints as each other.

The technique, together with the use of jumbles, lets numbers be represented by relatively few patterns, where each number is represented by one, and only one, pattern, and any two of the patterns contain different quantities of at least two letters or have opposite constraints on the letters. These patterns are the ‘basic patterns’ for a numbering space: each of the numbers in the numbering space can be represented by one, and only one, of the basic patterns.

The capacities of the basic patterns can be calculated by using the factors introduced in Section 4.1.3. The tables below illustrate this for the 10,000 four-digit numbers, the 100,000 five-digit numbers, the 1,000,000 six-digit numbers and the 10,000,000 seven-digit numbers. At the cost of being slightly more verbose than necessary they demonstrate the principles by which the patterns are formed (by providing all of the patterns as jumbles) and the overlap extents are devised (by including ‘0!’ and ‘1!’ in the letter order counts and overlap extents.

Table 45 Capacities of four-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a]	10!/9!	4!/(4!×0!)	1/0!	10
[a§a§a§b]	10!/8!	4!/(3!×1!)	1/1!	360
[a§a§b§b]	10!/8!	4!/(2!^2)	1/2!	270
[a§a§b§c]	10!/7!	4!/(2!×1!^2)	1/2!	4,320
[a§b§c§d]	10!/6!	4!/(1!^4)	1/4!	5,040

Table 46 Capacities of five-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a]	10!/9!	5!/(5!×0!)	1/0!	10
[a§a§a§a§b]	10!/8!	5!/(4!×1!)	1/1!	450
[a§a§a§b§b]	10!/8!	5!/(3!× 2!)	1/1!	900
[a§a§a§b§c]	10!/7!	5!/(3!×1!^2)	1/2!	7,200
[a§a§b§b§c]	10!/7!	5!/(2!^2×1!^1)	1/2!	10,800
[a§a§b§c§d]	10!/6!	5!/(2!×1!^3)	1/3!	50,400
[a§b§c§d§e]	10!/5!	5!/(1!^5)	1/5!	30,240

Table 47 Capacities of six-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§a]	10!/9!	6!/(6!×0!)	1/0!	10
[a§a§a§a§a§b]	10!/8!	6!/(5!×1!)	1/1!	540
[a§a§a§a§b§b]	10!/8!	6!/(4!× 2!)	1/1!	1,350
[a§a§a§b§b§b]	10!/8!	6!/(3!^2)	1/2!	900
[a§a§a§a§b§c]	10!/7!	6!/(4!×1!^2)	1/2!	10,800
[a§a§a§b§b§c]	10!/7!	6!/(3!×2!×1!^1)	1/1!	43,200
[a§a§b§b§c§c]	10!/7!	6!/(2!^3)	1/3!	10,800
[a§a§a§b§c§d]	10!/6!	6!/(3!×1!^3)	1/3!	100,800
[a§a§b§b§c§d]	10!/6!	6!/(2!^2×1!^2)	1/(2!×2!)	226,800
[a§a§b§c§d§e]	10!/5!	6!/(2!×1!^4)	1/4!	453,600
[a§b§c§d§e§f]	10!/4!	6!/(1!^6)	1/6!	151,200

Table 48 Capacities of seven-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§a§a]	10!/9!	7!/(7!×0!)	1/0!	10
[a§a§a§a§a§a§b]	10!/8!	7!/(6!×1!)	1/1!	630
[a§a§a§a§a§b§b]	10!/8!	7!/(5!× 2!)	1/1!	1,890
[a§a§a§a§b§b§b]	10!/8!	7!/(4!×3!)	1/1!	3,150
[a§a§a§a§a§b§c]	10!/7!	7!/(5!×1!^2)	1/2!	15,120
[a§a§a§a§b§b§c]	10!/7!	7!/(4!×2!×1!^1)	1/1!	75,600
[a§a§a§b§b§b§c]	10!/7!	7!/(3!^2×1!^1)	1/2!	50,400
[a§a§a§b§b§c§c]	10!/7!	7!/(3!×2!^2)	1/2!	75,600
[a§a§a§a§b§c§d]	10!/6!	7!/(4!×1!^3)	1/3!	176,400
[a§a§a§b§b§c§d]	10!/6!	7!/(3!×2!×1!^2)	1/2!	1,058,400
[a§a§b§b§c§c§d]	10!/6!	7!/(2!^3×1!^1)	1/3!	529,200
[a§a§a§b§c§d§e]	10!/5!	7!/(3!×1!^4)	1/4!	1,058,400
[a§a§b§b§c§d§e]	10!/5!	7!/(2!^2×1!^3)	1/(2!×3!)	3,175,200
[a§a§b§c§d§e§f]	10!/4!	7!/(2!×1!^5)	1/5!	3,175,200
[a§b§c§d§e§f§g]	10!/3!	7!/(1!^7)	1/7!	604,800

4.2 Pattern classification

4.2.1 Patterns in numbers

The notation devised in Section 4.1.2 permits the construction of patterns that have particular attractive features, such as repetitions, clusters and sequences. For the present purpose:

- Clusters are two parts of a pattern that are similar to each other in any of the following ways:
  - They must be identical with each other (as are '247' and '247' in '24782479')
  - They must be identical after reversing the digits in one of them (as are '247' and '742' in '24787429').
  - They must be identical after permuting the digits in one of them (as are '247' and '274' in '24782749').
- Sequences, which are formed from consecutive digits such as '0', '1', '2', … or '2', '3', '4', have rather unusual forms here: the digits in a sequence may be repeated and permuted (so '01243567', '01234576' and '01234566' are sequences). To establish a clear demarcation between a sequence having this form and the remainder of the pattern in which it appears, one of the following conditions must hold:

  - All of the occurrences of each digit in the pattern are in the sequence (as in '12233344' and '14132332').
  - All of the occurrences of digits that occur more than once in the pattern are shared equally between clusters (as in '82349234' and '82349432').

Care is needed when determining the capacities of patterns that include clusters, as a cluster might include another one. For instance:

- For three-digit clusters:
  - The three-digit clusters identified by '[a§a§b][a§a§b]' always include two-digit clusters ('aababa', for example).
  - The three-digit clusters identified by '[a§b§c][a§b§c]' always include two-digit clusters ('abcbac', for example).
- For four-digit clusters:
  - The four-digit clusters identified by '[a§a§a§b][a§a§a§b]' always include three-digit clusters ('aaababaa', for example).
  - The four-digit clusters identified by '[a§a§b§b][a§a§b§b]' always include two-digit clusters but do not always include three-digit clusters ('abbabaab', for example).
  - The four-digit clusters identified by '[a§a§b§c][a§a§b§c]' always include two-digit clusters but do not always include three-digit clusters ('abcabaac', for example).
  - The four-digit clusters identified by '[a§b§c§d][a§b§c§d]' do not always include two-digit clusters ('abcddbac', for example).

The capacities of patterns that include sequences are discussed for the sequences considered here in Section 4.2.3. More usual sequences are examined in Section 5.2.3.

4.2.2 Composition of patterns in the numbering space

The techniques developed in Section 4.1.3 can be applied to determine the capacities of patterns that represent all of the numbers in the numbering space and that differ from each other in their arrangements of digits. The preferred arrangements of digits are those relating to repetitions, clusters and sequences, which are assumed to affect the attractiveness of the patterns.

To determine appropriate patterns, the complete eight-digit numbering space, containing every eight-digit number, is split thus:

- Patterns that represent all arrangements of digits are devised, with one pattern per arrangement. Each eight-digit number is represented by one, and only one, of these patterns, so the capacities of the patterns sum to

100,000,000. There are twenty-two of these basic patterns, with capacities ranging between ten (when there are eight identical digits) and 31,752,000 (when there are six different digits, two of which occur twice each). The patterns are those provided in Section 4.4.1.

- Patterns that include clusters are devised. To qualify as clusters, two parts of a pattern must be similar to each other, in the sense described in Section 4.2.1. The patterns are those provided in Section 4.4.2.
- Patterns that include clusters are separated out from patterns that represent all arrangements of digits if they have much smaller capacities than the ones from which they are separated (typically by a factor of at least five). In practice this entails separating out the patterns where the clusters must be identical and patterns where the clusters may be any permutations of each other, but not patterns where the clusters must be identical or reversed (because there are only twice as many of them as there are patterns where the clusters must be identical). The patterns are those provided in Section 4.4.3. Some of the original patterns are thereby split into several others; for instance, separating out the patterns that include clusters from the remainder of the pattern that has six different digits, two of which occur twice each, reduces its capacity from 31,752,000 to 27,216,000.
- Patterns that include sequences are separated out from the patterns that represent all arrangements of digits. Each such pattern contains only digits from one sequence (and all digits in the pattern are in the sequence), but a digit might occur more than once. The digits in the sequence might be repeated and permuted; for instance, '22223345' and '22522343' are separated out from '22722393'. The capacities of these patterns can be obtained from the capacities of the corresponding patterns in Section 4.4.1 as illustrated in Section 4.2.3.
- Patterns that include clusters formed from sequences are separated out. Each such pattern contains only digits from one sequence (and some digits in the pattern might not be in the sequence), but a digit might occur more than once. The digits in the sequence can be in any order; for instance, '12379123' and '13279132' are separated out from '12479124'. For these clusters again the capacities can be obtained from the capacities of the corresponding patterns in Section 4.4.2 as illustrated in Section 4.2.3.

The patterns that result from splitting the numbering space in this way place all the numbers in separate classes.

4.2.3 Capacities for sequences

Capacities need to be calculated for the patterns that result from splitting the numbering space in the way outlined in Section 4.2.2. Separate digit choice counts, letter order counts and overlap extents can help with doing this, because they can be adjusted independently of each other. In particular, digit choice counts can be scaled.

An illustration is provided by patterns that include sequences, which are formed from consecutive digits such as '0', '1', '2', … or '2', '3', '4', …. Here, partly to illustrate the scaling of digit choice counts, sequences have rather unusual forms: the digits in a sequence may be repeated and permuted (so '01243567', '01234576' and '01234566' are sequences here). A clear demarcation between a sequence and the remainder of the pattern in which it appears can be established by defining sequences as in Section 4.4.1.

The capacity of a pattern that is constrained to include a sequence of this sort can be obtained readily from the capacity of the corresponding pattern that is not constrained in this way. All that needs to be done is to change the digit choice count. For instance:

- In Section 4.4.1, if the pattern contains occurrences of four letters, then the digit choice count is 10!/6!, because there are ten ways of choosing a first digit, nine ways of choosing a second, eight ways of choosing a third and seven ways of choosing a fourth. The digit choice count for choosing all of the digits in the pattern to be a sequence is 4!×7!/6! (or 4!×7), because there are seven sequences of four consecutive digits lying between '0' and '9', and distinct digits can be represented by any of the four letters.

- In Section 4.4.2, if the pattern contains occurrences of five letters and includes clusters that contain three distinct letters), then the digit choice count is 10!/5!. The digit choice count for choosing just the digits in the cluster to be a sequence is 3!×8!/5! (or 3!×8×7!/5!), because there are eight sequences of three consecutive digits lying between '0' and '9', there are two further digits to be chosen outside the clusters, and distinct digits in the clusters can be represented by any of the three letters. In this case the letters in the cluster are constrained differently from the other letters, so there must be no occurrences of those letters outside the cluster.

- In Section 4.4.3, the patterns that include clusters are separated out from the patterns that represent all arrangements of digits. Along with them are exceptions representing the arrangements of digits that do not include clusters.

- In Section 4.4.4, the clusters that are formed from sequences are singled out according to the following principles:

- Clusters that are formed from sequences are regarded as not being made more attractive by having letters outside them that extend the sequences.
- Clusters that contain occurrences of only one letter are not recognised as being formed from sequences.

The technique of scaling digit choice counts can be applied to other features of numbers that could be attractive, besides sequences. For instance, it can applied to find the proportion of patterns in which favoured digits such as '0' and '5' occur frequently (where clusters containing occurrences of only one letter could be regarded as representing favoured digits though they could not be regarded as being formed from sequences).

4.2.4 Tables for the classification

The basic patterns for the complete eight-digit numbering space, containing every eight-digit number, are set out in Section 4.4.1. These patterns provide a foundation for an analysis of the repetitions, clusters and sequences in the numbers. An example of this is in Section 4.4.5, where the capacities of patterns containing repetitions are calculated. However, the main application in this report is to clusters and sequences.

Clusters defined as in Section 4.2.1 are tabulated in Section 4.4.2. The tables there distinguish between clusters according to how many occurrences of distinct digits they contain. The clusters are collected under their basic patterns in Section 4.4.3, to illustrate how they are separated out from the remainders of the basic patterns. They are then split further, into patterns comprising sequences and patterns lacking sequences, by calculating the capacities of these patterns using the principles of Section 4.2.2.

The resulting tables, in Section 4.4.4, are merged and sorted into bands (A, B, C, D, E, F, G, H, I, J, K and L) in Section 4.4.6, using the technique for defining bands given in Section 4.3.1.

4.2.5 Limitations of the classification

The classification of numbers summarised in Section 4.2.4 has several good points: it is succinct, it provides general reasoning about sets of patterns, it separates different factors in the calculation of capacities, and it provides formulas relating capacities to pattern structure.

However, in its most straightforward form, it considers patterns to be attractive only if they use either all or none of their occurrences of each letter. It is satisfactory for patterns that have few distinct letters, because such patterns are rare and always appear attractive for one reason or another. It is rather less satisfactory when patterns have several distinct digits, because it omits from the attractive patterns some that might be regarded as including repetitions (such as '22249257'), clusters (such as '27492757') or sequences (such as '12345757').

Ultimately these omissions might not matter, as such patterns are generally not very rare and therefore probably not very attractive; for instance, in the complete eight-digit numbering space, patterns that are attractive only by having three-digit repetitions constitute more than 3% and patterns that are attractive only by having two-digit clusters constitute more than 8%. In addition, the omissions might not matter if the numbers are finally put into very few price bands: numbers having these patterns will be very cheap. Nonetheless some concern about the omissions is justified.

The formulas implicit in Section 4.4.2 can be extended in various ways to accommodate omitted patterns, such as patterns in which some, but not all, of the occurrences of a digit contribute to the repetitions, clusters and sequences. However, the extensions are not entirely straightforward to devise (as in effect they modify the algebra of patterns so that some patterns commute with others). Moreover, the extensions become ever more difficult to deploy when patterns that have two or three attractive features are to be separated out from patterns that have fewer attractive features. Separating out these patterns is essential if the patterns are to be placed in many different classes before being ranked.

Consequently a different classification of numbers is developed in Section 5.2.4.

4.3 Number banding

4.3.1 Determination of bands from capacities

The patterns tabulated in Section 4.4.4 show how numbers composed in the same way as one another can be put in the same class. However, there are too many classes for an overview. Here the patterns are put into rather fewer bands, which are determined by the capacities of the patterns.

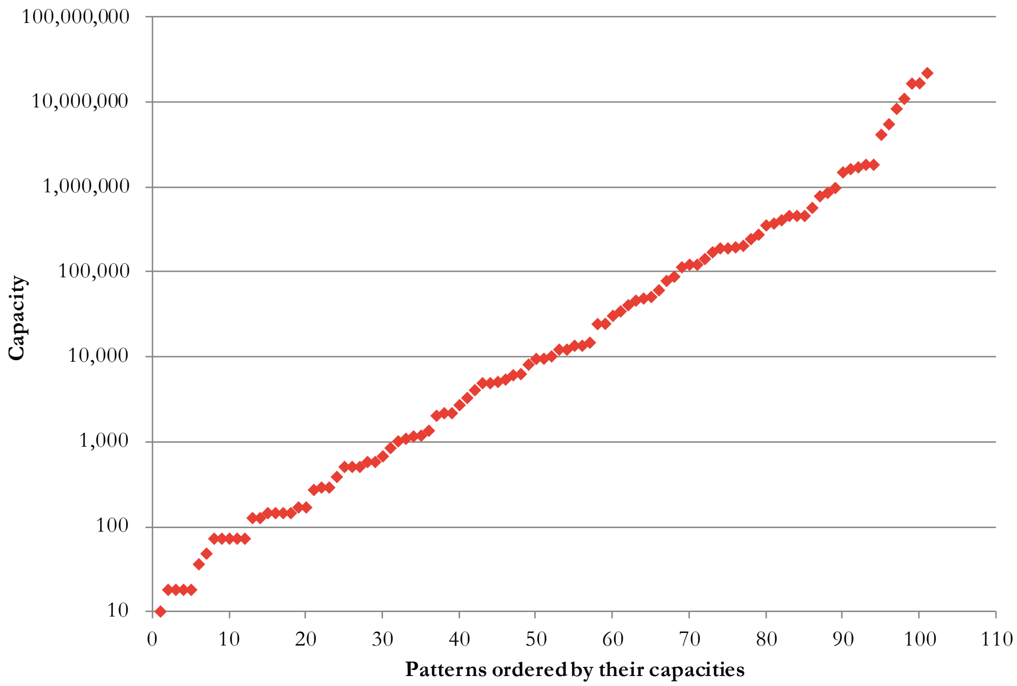
Table 49 puts in ranges with different capacities the patterns tabulated in Section 4.4.6 (which are those of Section 4.2.4 refined by separating out the patterns that include sequences). There are roughly the same quantities of patterns in four of the ranges, but the bottom range and the top two ranges are rather smaller.

Table 49 Distribution of eight-digit patterns in the complete eight-digit numbering space with clusters and sequences separated out

Capacity	Quantity of patterns	Proportion of patterns (%)	Quantity of numbers	Proportion of numbers (%)
10-99	12	11.88	526	0.0005
100-999	19	18.81	6,570	0.0066
1,000-9,999	20	19.80	81,456	0.0815
10,000-99,999	17	16.83	599,256	0.5993
100,000-999,999	21	20.79	7,588,224	7.5882
1,000,000-9,999,999	8	7.92	26,212,032	26.2120
10,000,000-99,999,999	4	3.96	65,511,936	65.5119
10-99,999,999	101	100.00	100,000,000	100.0000

Figure 1 shows that these patterns are distributed fairly evenly throughout these ranges, by plotting the capacities on a logarithmic scale. However, there are some gaps in the graph that could be regarded as gaps between bands.

Figure 1 Distribution of eight-digit patterns in the complete eight-digit numbering space with clusters and sequences separated out



The table in Section 4.4.6 demonstrates the effect of taking a gap to exist when the capacity of one pattern is at least half as much again as the capacity of the next most capacious pattern. The patterns are put in bands (A, B, C, D, E, F, G, H, I, J, K and L) bounded by the gaps. The table also provides examples of the numbers represented by the patterns.

4.3.2 Limitations of the banding

The investigation above indicates that, when patterns are ranked according to their capacities, there could be gaps in the ranking that might justify different bands. However, the following arguments suggest that the usefulness of this result is limited:

- The ranking depends for its appropriateness on determining all of the attractive features to the same level of detail. Only then can capacity represent rarity fairly.

- Separating out from a class the patterns that have attractive features can leave a remainder without attractive features but that has a smaller capacity than the patterns with attractive features. In such cases, unless extra discriminants are introduced, the remainder will be ranked higher than the patterns with attractive features.

- Even when all of the attractive features are separated out to the same level of detail, some distinctions between the bands can be questioned. For instance, in the table in Section 4.4.6, the numbers in band B are not obviously much more attractive than their counterparts in band D (perhaps because, in this case, the distinctions overstate the attractiveness of sequences relative to repetitions).

- Advertisements for special numbers often lay stress on rarity, in the belief that customers appreciate rarity. However, customers do not have any detailed knowledge of how rare different special numbers are; if they assess rarity at all, they do so by looking at the features of the numbers. For instance, three-digit repetitions (with or without other attractive features) constitute about 5% of the numbering space, but customers might imagine them to be rare.

- There could be too many or too few gaps in the ranking of capacities. Indeed, the table in Section 4.2.6 still has twelve bands, if a gap is taken to exist when the capacity of one pattern is at least half as much again as the capacity of the next most capacious pattern. Similarly, though only six bands results from a comparable investigation of the patterns devised using the classification of numbers summarised in Section 5.2.4, the first three bands together contain 794 numbers while the fourth band alone contains 52,731,860; in fact the absence of gaps across half the numbering space is caused by the very similar pattern capacities and could be evidence that the classification determines all of the attractive features to the same level of detail.

There are variants of the investigation above that are still based on capacities but that diminish the force of some of these arguments. In particular, the criterion for identifying a gap could depend not just whether the capacity of one pattern exceeds the capacity of one less capacious pattern by some factor, but on whether the capacity of one pattern exceeds a weighted sum of the capacities of several less capacious patterns; however, though there might be a justification from psychology for doing this (perhaps by analogy with hyperbolic discounting) the weights are arbitrary and, again, lead to too many or too few gaps. Overall, then, bands are not determined satisfactorily by pattern capacities.

4.4 Analysis of the eight-digit numbering space

4.4.1 Capacities for patterns representing each number in the complete eight-digit numbering space once only

Table 50 Capacities of eight-digit patterns containing occurrences of exactly one letter

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§a§a§a]	10!/9!	8!/(8!×1!^0)	1/0!	10

Table 51 Capacities of eight-digit patterns containing occurrences of exactly two letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§a§a§b]	10!/8!	8!/(7!×1!^1)	1/1!	720
[a§a§a§a§a§a§b§b]	10!/8!	8!/(6!×2!^1!^0)	1/0!	2,520
[a§a§a§a§a§b§b§b]	10!/8!	8!/(5!×3!×1!^0)	1/0!	5,040
[a§a§a§a§b§b§b§b]	10!/8!	8!/(4!^2×1!^0)	1/(2!×0!)	3,150

Table 52 Capacities of eight-digit patterns containing occurrences of exactly three letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§a§b§c]	10!/7!	8!/(6!×1!^2)	1/2!	20,160
[a§a§a§a§a§b§b§c]	10!/7!	8!/(5!×2!×1!^1)	1/1!	120,960
[a§a§a§a§b§b§b§c]	10!/7!	8!/(4!×3!×1!^1)	1/1!	201,600
[a§a§a§a§b§b§c§c]	10!/7!	8!/ (4!×2!^2×1!^0)	1/(2!×0!)	151,200
[a§a§a§b§b§b§c§c]	10!/7!	8!/(3!^2×2!^1)	1/(2!×0!)	201,600

Table 53 Capacities of eight-digit patterns containing occurrences of exactly four letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§b§c§d]	10!/6!	8!/(5!×1!^3)	1/3!	282,240
[a§a§a§a§b§b§c§d]	10!/6!	8!/(4!×2!×1!^2)	1/2!	2,116,800
[a§a§a§b§b§b§c§d]	10!/6!	8!/(3!^2×1!^2)	1/(2!×2!)	1,411,200
[a§a§a§b§b§c§c§d]	10!/6!	8!/(3!×2!^2×1!^1)	1/(2!×1!)	4,233,600
[a§a§b§b§c§c§d§d]	10!/6!	8!/(2!^4)	1/4!	529,200

Table 54 Capacities of eight-digit patterns containing occurrences of exactly five letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§b§c§d§e]	10!/5!	8!/(4!×1!^4)	1/4!	2,116,800
[a§a§a§b§b§c§d§e]	10!/5!	8!/(3!×2!×1!^3)	1/3!	16,934,400
[a§a§b§b§c§c§d§e]	10!/5!	8!/(2!^3×1!^2)	1/(3!×2!)	12,700,800

Table 55 Capacities of eight-digit patterns containing occurrences of exactly six letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§b§c§d§e§f]	10!/4!	8!/(3!×1!^5)	1/5!	8,467,200
[a§a§b§b§c§d§e§f]	10!/4!	8!/(2!^2×1!^4)	1/4!	31,752,000

Table 56 Capacities of eight-digit patterns containing occurrences of exactly seven letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§b§c§d§e§f§g]	10!/3!	8!/(2!×1!^6)	1/6!	16,934,400

Table 57 Capacities of eight-digit patterns containing occurrences of exactly eight letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§b§c§d§e§f§g§h]	10!/2!	8!/(1!^8)	1/8!	1,814,400

4.4.2 Capacities for patterns distinguishing clusters

Table 58 Capacities of eight-digit patterns containing occurrences of exactly two letters, with four-digit clusters containing occurrences of exactly two letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaab§aaab]	10!/8!	2!/2!	1/1!	90
[aaab§baaa]	10!/8!	2!/1!	1/1!	180
[[a§a§a§b]§[a§a§a§b]]	10!/8!	2!/2!×(4!/3!)^2	1/1!	1,440
[aaaa§bbbb]	10!/8!	2!/1!	1/2!	90
[aabb§aabb]	10!/8!	2!/2!	1/1!	90
[aabb§bbaa]	10!/8!	2!/1!	1/2!	90
[[a§a§b§b]§[a§a§b§b]]	10!/8!	2!/2!× (4!/(2!×2!))^2	1/2!	1,620

Table 59 Capacities of eight-digit patterns containing occurrences of exactly two letters, with two-digit clusters containing occurrences of exactly one letter

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aa§aa§aa§bb]	10!/8!	4!/(3!×1!^1)	1/1!	360
[aa§aa§bb§bb]	10!/8!	4!/(2!^2)	1/2!	270

Table 60 Capacities of eight-digit patterns containing occurrences of exactly two letters, with two-digit clusters containing occurrences of exactly two letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[ab§ab§ab§ab]	10!/8!	4!/(4!×1!^0)	1/0!	90
[[a§b]§[a§b]§ [a§b]§[a§b]]	10!/8!	4!/(4!×1!^0)× 2!^4	1/2!	720

Table 61 Capacities of eight-digit patterns containing occurrences of exactly three letters, with four-digit clusters containing occurrences of exactly three letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aabc§aabc]	10!/7!	2!/2!	1/1!	720
[aabc§cbaa]	10!/7!	2!/1!	1/1!	1,440
[[aa§[b§c]]§[aa§[b§c]]]	10!/7!	2!/2!×(1!×2!)^2	1/2!	2,880
[[a§a§b§c]§[a§a§b§c]]	10!/7!	2!/2!×(4!/2!)^2	1/2!	51,840

Table 62 Capacities of eight-digit patterns containing exactly three letters, with three-digit clusters containing occurrences of exactly one letter

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaa§aaa§b§c]	10!/7!	4!/(2!×1!^2)	1/2!	4,320

Table 63 Capacities of eight-digit patterns containing occurrences of exactly three letters, with two-digit clusters containing occurrences of exactly one letter

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aa§aa§aa§bc]	10!/7!	4!/(3!×1!^1)	1/1!	2,880
[aa§aa§aa§[b§c]]	10!/7!	4!/(3!×1!^2)×2!	1/2!	2,880
[aa§aa§aa§b§c]	10!/7!	5!/(3!×1!^2)	1/2!	7,200
[aa§aa§bc§bc]	10!/7!	4!/(2!×2!)	1/0!	4,320
[aa§aa§bc§cb]	10!/7!	4!/(2!×1!×1!)	1/2!	4,320
[aa§aa§[b§c]§[b§c]]	10!/7!	4!/(2!×2!)×2!^2	1/2!	8,640

Table 64 Capacities of eight-digit patterns containing occurrences of exactly four letters, with four-digit clusters containing occurrences of exactly four letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[abcd§abcd]	10!/6!	2!/2!	1/1!	5,040
[abcd§dcba]	10!/6!	2!/1!	1/2!	5,040
[[[a§b]§[c§d]]§ [[a§b]§ [c§d]]]	10!/6!	2!/2!× (2!×2!^2)^2	1/ (2!×2!×2!)	40,320
[[a§b§c§d]§[a§b§c§d]]	10!/6!	2!/2!×4!^2	1/(4!×1!)	120,960

Table 65 Capacities of eight-digit patterns containing occurrences of exactly four letters, with three-digit clusters containing occurrences of exactly two letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aab§aab§c§d]	10!/6!	4!/(2!×1!^2)	1/2!	30,240
[aab§baa§c§d]	10!/6!	4!/(1!×1!×1!^2)	1/(1!×2!)	60,480
[[a§a§b]§[a§a§b]§c§d]	10!/6!	4!/(2!×1!^2)× (3!/2!)^2	1/2!	272,160

Table 66 Capacities of eight-digit patterns containing occurrences of exactly four letters, with two-digit clusters containing occurrences of exactly two letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[ab§ab§ab§cd]	10!/6!	4!/(3!×1!^1)	1/1!	20,160
[ab§ab§ba§cd]	10!/6!	4!/(2!×1!×1!^1)	1/1!	60,480
[ab§ab§ab§c§d]	10!/6!	5!/(3!×1!^2)	1/2!	50,400
[ab§ab§ba§c§d]	10!/6!	5!/(2!×1!×1!^2)	1/2!	151,200
[[a§b]§[a§b]§[a§b]§c§d]	10!/6!	5!/(3!×1!^2)× 2!^3	1/(2!×2!)	201,600
[abab§cdcd]	10!/6!	2!/1!	1/2!	5,040
[abab§cddc]	10!/6!	2!/1!	1/1!	10,080
[abba§cddc]	10!/6!	2!/1!	1/2!	5,040
[[a§b]§[a§b]]§ [[c§d]§[c§d]]	10!/6!	2!/1!× (2!/2!×2!^2)^2	1/ (2!×2!×2!)	20,160
[[a§a§b§b]§[c§c§d§d]]	10!/6!	2!/1!× (4!/(2! ×2!))^2	1/ (2!×2!×2!)	45,360

Table 67 Capacities of eight-digit patterns containing occurrences of exactly five letters, with three-digit clusters containing occurrences of exactly three letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[abc§abc§d§e]	10!/5!	4!/(2!×1!^2)	1/2!	181,440
[abc§cba§d§e]	10!/5!	4!/(1!×1!×1!^2)	1/(2!×2!)	181,440
[[a§b§c]§[a§b§c]§d§e]	10!/5!	4!/(2!×1!^2)× 3!^2	1/(3!×2!)	1,088,640

Table 68 Capacities of eight-digit patterns containing occurrences of exactly five letters, with two-digit clusters containing occurrences of exactly one letter

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aa§aa§b§c§d§e]	10!/5!	6!/(2!×1!^4)	1/4!	453,600

Table 69 Capacities of eight-digit patterns containing occurrences of exactly six letters, with two-digit clusters containing occurrences of exactly two letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[ab§ab§c§d§e§f]	10!/4!	6!/(2!×1!^4)	1/4!	2,268,000
[[a§b]§[a§b]§c§d§e§f]	10!/4!	6!/(2!×1!^4)× 2!^2	1/(2!×4!)	4,536,000

4.4.3 Capacities for patterns separating clusters to avoid overlaps

Table 70 Capacities of eight-digit patterns containing occurrences of exactly one letter

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§a§a§a]	10!/9!	8!/(8!×1!^0)	1/0!	10

Table 71 Capacities of eight-digit patterns containing occurrences of exactly two letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§a§a§b]	10!/8!	8!/(7!×1!^1)	1/1!	720
[a§a§a§a§a§b§b§b]	10!/8!	8!/(5!×3!×1!^0)	1/0!	5,040
[aaab§aaab]	10!/8!	2!/2!	1/1!	90
[[a§a§a§b]§[a§a§a§b]]\ [aaab§aaab]	10!/8!			1,350
[aa§aa§aa§bb]	10!/8!	4!/(3!×1!^1)	1/1!	360
[a§a§a§a§a§b§b]\ [[a§a§a§b]§[a§a§a§b]]\ [aa§aa§aa§bb]	10!/8!			720
[aaaa§bbbb]	10!/8!	2!/1!	1/2!	90
[aabb§aabb]	10!/8!	2!/2!	1/1!	90
[aa§aa§bb§bb]\ [aabb§aabb]	10!/8!			180

Formulating patterns

[ab§ab§ab§ab]	10!/8!	4!/(4!×1!^0)	1/0!	90
[[a§b]§[a§b]§[a§b]§ [a§b]]\ [ab§ab§ab§ab]	10!/8!			630
[[a§a§b§b]§[a§a§b§b]]\ [aa§aa§bb§bb]\ [[a§b]§[a§b]§[a§b]§ [a§b]]	10!/8!			630
[a§a§a§a§b§b§b§b]\ [[a§a§b§b]§[a§a§b§b]]\ [aaaa§bbbb]	10!/8!			1,440

Table 72 Capacities of eight-digit patterns containing occurrences of exactly three letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§b§b§c]	10!/7!	8!/(5!×2!×1!^1)	1/1!	120,960
[a§a§a§a§b§b§b§c]	10!/7!	8!/(4!×3!×1!^1)	1/1!	201,600
[a§a§a§b§b§b§c§c]	10!/7!	8!/(3!^2×2!^1)	1/(2!×0!)	201,600
[aaaaaa§b§c]	10!/7!	3!/(1!×1!^2)	1/2!	2,160
[aaa§aaa§b§c]\ [aaaaaa§b§c]	10!/7!			2,160
[aa§aa§aa§b§c]\ [aaaaaa§b§c]	10!/7!			5,040
[a§a§a§a§a§a§b§c]\ [aaa§aaa§b§c]\ [aa§aa§aa§b§c]	10!/7!			10,800
[aabc§aabc]	10!/7!	2!/2!	1/1!	720
[[aa§[b§c]]§[aa§[b§c]]]\ [aabc§aabc]	10!/7!			2,160
[[a§a§b§c]§[a§a§b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	10!/7!			48,960
[aa§aa§[b§c]§[b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	10!/7!			5,760
[a§a§a§a§b§b§c§c]\ [[a§a§b§c]§[a§a§b§c]]\ [aa§aa§[b§c]§[b§c]]	10!/7!			93,600

Table 73 Capacities of eight-digit patterns containing occurrences of exactly four letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§a§a§b§c§d]	10!/6!	8!/(5!×1!^3)	1/3!	282,240
[a§a§a§b§b§c§c§d]	10!/6!	8!/ (3!×2!^2×1!^1)	1/(2!×1!)	4,233,600
[aab§aab§c§d]	10!/6!	4!/(2!×1!^2)	1/2!	30,240
[[a§a§b]§[a§a§b]§c§d]\ [aab§aab§c§d]	10!/6!			241,920
[a§a§a§a§b§b§c§d]\ [[a§a§b]§[a§a§b]§c§d]	10!/6!			1,844,640
[ab§ab§ab§c§d]	10!/6!	5!/(3!×1!^2)	1/2!	50,400
[[a§b]§[a§b]§[a§b]§c§d] \[ab§ab§ab§c§d]	10!/6!			151,200
[a§a§a§b§b§b§c§d]\ [[a§b]§[a§b]§[a§b]§c§d]	10!/6!			1,209,600
[abcd§abcd]	10!/6!	2!/2!	1/1!	5,040
[[[a§b]§[c§d]]§ [[a§b]§ [c§d]]]\ [abcd§abcd]	10!/6!			35,280
[[a§b§c§d]§[a§b§c§d]]\ [[[[a§b]§[c§d]]§ [[a§b]§ [c§d]]]	10!/6!			80,640
[abab§cdcd]	10!/6!	4!/ 1!	1/2!	5,040
[[[a§b]§[a§b]]§ [[c§d]§ [c§d]]]\ [abab§cdcd]	10!/6!			15,120
[[a§a§b§b]§[c§c§d§d]]\ [[[a§b]§[a§b]]§ [[c§d]§ [c§d]]]	10!/6!			25,200
[a§a§b§b§c§c§d§d]\ [[a§b§c§d]§[a§b§c§d]]\ [[a§a§b§b]§[c§c§d§d]]	10!/6!			362,880

Table 74 Capacities of eight-digit patterns containing occurrences of exactly five letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§b§b§c§d§e]	10!/5!	8!/(3!×2!×1!^3)	1/3!	16,934,400
[aa§aa§b§c§d§e]	10!/5!	6!/(2!×1!^4)	1/4!	453,600
[a§a§a§a§b§c§d§e]\ [aa§aa§b§c§d§e]	10!/5!			1,663,200
[abc§abc§d§e]	10!/5!	4!/(2!×1!^2)	1/2!	181,440
[[a§b§c]§[a§b§c]§d§e]\ [abc§abc§d§e]	10!/5!			907,200
[a§a§b§b§c§c§d§e]\ [[a§b§c]§[a§b§c]§d§e]	10!/5!			11,612,160

Table 75 Capacities of eight-digit patterns containing occurrences of exactly six letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§a§b§c§d§e§f]	10!/4!	8!/(3!×1!^5)	1/5!	8,467,200
[ab§ab§c§d§e§f]	10!/4!	6!/(2!×1!^4)	1/4!	2,268,000
[[a§b]§[a§b]§c§d§e§f]\ [ab§ab§c§d§e§f]	10!/4!			2,268,000
[a§a§b§b§c§d§e§f]\ [[a§b]§[a§b]§c§d§e§f]	10!/4!			27,216,000

Table 76 Capacities of eight-digit patterns containing occurrences of exactly seven letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§a§b§c§d§e§f§g]	10!/3!	8!/(2!×1!^6)	1/6!	16,934,400

Table 77 Capacities of eight-digit patterns containing occurrences of exactly eight letters

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§b§c§d§e§f§g§h]	10!/2!	8!/(1!^8)	1/8!	1,814,400

4.4.4 Capacities for patterns representing each number in the complete eight-digit numbering space once only and separating clusters to avoid overlaps

Table 78 Capacities of eight-digit patterns containing occurrences of exactly one letter

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§a§a§a§a§a§a§a]	10		10

Table 79 Capacities of eight-digit patterns containing occurrences of exactly two letters

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§a§a§a§a§a§a§b]	720	144	576
[a§a§a§a§a§b§b§b]	5,040	1,008	4,032
[aaab§aaab]	90	18	72
[[a§a§a§b]§[a§a§a§b]]\ [aaab§aaab]	1,350	270	1,080
[aa§aa§aa§bb]	360	72	288
[a§a§a§a§a§a§b§b]\ [[a§a§a§b]§[a§a§a§b]]\ [aa§aa§aa§bb]	720	144	576
[aaaa§bbbb]	90	18	72
[aabb§aabb]	90	18	72
[aa§aa§bb§bb]\ [aabb§aabb]	180	36	144
[ab§ab§ab§ab]	90	18	72
[[a§b]§[a§b]§ [a§b]§[a§b]]\ [ab§ab§ab§ab]	630	126	504
[[a§a§b§b]§[a§a§b§b]]\ [aa§aa§bb§bb]\ [[a§b]§[a§b]§ [a§b]§[a§b]]	630	126	504
[a§a§a§a§b§b§b§b]\ [[a§a§b§b]§[a§a§b§b]]\ [aaaa§bbbb]	1,440	288	1,152

Table 80 Capacities of eight-digit patterns containing occurrences of exactly three letters

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§a§a§a§a§b§b§c]	120,960	8,064	112,896
[a§a§a§a§b§b§b§c]	201,600	13,440	188,160
[a§a§a§b§b§b§c§c]	201,600	13,440	188,160
[aaaaaa§b§c]	2,160		2,160
[aaa§aaa§b§c]\ [aaaaaa§b§c]	2,160		2,160
[aa§aa§aa§b§c]\ [aaaaaa§b§c]	5,040		5,040
[a§a§a§a§a§a§b§c]\ [aaa§aaa§b§c]\ [aa§aa§aa§b§c]	10,800		9,456
[aabc§aabc]	720	48	672
[[aa§[b§c]]§[aa§[b§c]]]\ [aabc§aabc]	2,160	144	2,016
[[a§a§b§c]§[a§a§b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	48,960	3,264	45,696
[aa§aa§[b§c]§[b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	5,760	384	5,376
[a§a§a§a§b§b§c§c]\ [[a§a§b§c]§[a§a§b§c]]\ [aa§aa§[b§c]§[b§c]]	93,600	6,240	87,360

Table 81 Capacities of eight-digit patterns containing occurrences of exactly four letters

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§a§a§a§a§b§c§d]	282,240	9,408	272,832
[a§a§a§b§b§c§c§d]	4,233,600	141,120	4,092,480
[aab§aab§c§d]	30,240	6,048	24,192
[[a§a§b]§[a§a§b]§c§d]\ [aab§aab§c§d]	241,920	48,384	193,536
[a§a§a§a§b§b§c§d]\ [[a§a§b]§[a§a§b]§c§d]	1,844,640	368,928	1,475,712
[ab§ab§ab§c§d]	50,400	10,080	40,320
[[a§b]§[a§b]§ [a§b]§c§d]\ [ab§ab§ab§c§d]	151,200	30,240	120,960
[a§a§a§b§b§b§c§d]\ [[a§b]§[a§b]§[a§b]§c§d]	1,209,600	241,920	967,680

[abcd§abcd]	5,040	168	4,872
[[[a§b]§[c§d]]§ [[a§b]§ [c§d]]]\ [abcd§abcd]	35,280	1,176	34,104
[[a§b§c§d]§[a§b§c§d]]\ [[[[a§b]§[c§d]]§ [[a§b]§ [c§d]]]	80,640	2,688	77,952
[abab§cdcd]	5,040	168	4,872
[[[a§b]§[a§b]]§ [[c§d]§ [c§d]]]\ [abab§cdcd]	15,120	504	14,616
[[a§a§b§b]§[c§c§d§d]]\ [[[a§b]§[a§b]]§ [[c§d]§ [c§d]]]	25,200	840	24,360
[a§a§b§b§c§c§d§d]\ [[a§b§c§d]§[a§b§c§d]\ [a§a§b§b]§[c§c§d§d]]	362,880	12,096	350,784

Table 82 Capacities of eight-digit patterns containing occurrences of exactly five letters

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§a§a§b§b§c§d§e]	16,934,400	403,200	16,531,200
[aa§aa§b§c§d§e]	453,600		453,600
[a§a§a§a§b§c§d§e]\ [aa§aa§b§c§d§e]	1,663,200		1,612,800
[abc§abc§d§e]	181,440	12,096	169,344
[[a§b§c]§[a§b§c]§d§e]\ [abc§abc§d§e]	907,200	60,480	846,720
[a§a§b§b§c§c§d§e]\ [[a§b§c]§[a§b§c]§d§e]	11,612,160	774,144	10,838,016

Table 83 Capacities of eight-digit patterns containing occurrences of exactly six letters

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§a§a§b§c§d§e§f]	8,467,200	201,600	8,265,600
[ab§ab§c§d§e§f]	2,268,000	453,600	1,814,400
[[a§b]§[a§b]§c§d§e§f]\ [ab§ab§c§d§e§f]	2,268,000	453,600	1,814,400
[a§a§b§b§c§d§e§f]\ [[a§b]§[a§b]§c§d§e§f]	27,216,000	5,443,200	21,772,800

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Table 84 Capacities of eight-digit patterns containing occurrences of exactly seven letters

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§a§b§c§d§e§f§g]	16,934,400	564,480	16,369,920

Table 85 Capacities of eight-digit patterns containing occurrences of exactly eight letters

Pattern	Capacity for full coverage	Capacity limited to sequences	Capacity remaining
[a§b§c§d§e§f§g§h]	1,814,400	120,960	1,693,440

4.4.5 Capacities for patterns distinguishing repetitions

Table 86 Capacities of eight-digit patterns containing occurrences of exactly one letter, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaaaaaaa]	10!/9!	8!/(8!×1!^0)	1/0!	10

Table 87 Capacities of eight-digit patterns containing occurrences of exactly two letters, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaaaaaa§b]	10!/8!	2!/(1!×1!^1)	1/0!	180
[aaaaaa§b§b]	10!/8!	3!/(1!×2!^1)	1/1!	270
[a§a§a§a§a§a§bb]	10!/8!	7!/(6!×1!)	1/0!	630
[aaaaaa§bb]	10!/8!	2!/(1!×1!)	1/0!	180
[aaaaa§b§b§b]	10!/8!	4!/(1!×3!^1)	1/1!	360
[a§a§a§a§a§bbb]	10!/8!	6!/(5!×1!)	1/0!	540
[aaaaa§bbb]	10!/8!	3!/(1!×1!)	1/0!	180
[aaaa§b§b§b§b]	10!/8!	5!/(1! ×4!^1)	1/1!	450
[aaaa§bbbb]	10!/8!	2!/(1!^2)	1/2!	90

Table 88 Capacities of eight-digit patterns containing occurrences of exactly three letters, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaaaaa§b§c]	10!/7!	3!/(1!×1!^2)	1/2!	2,160
[aaaaa§b§b§c]	10!/7!	4!/(1!×2!×1!^1)	1/1!	8,640
[a§a§a§a§a§bb§c]	10!/7!	7!/(5!×1!×1!^1)	1/1!	30,240
[aaaaa§bb§c]	10!/7!	3!/(1!×1!×1!^1)	1/1!	4,320
[aaaa§b§b§b§c]	10!/7!	5!/(1!×3!×1!^1)	1/1!	14,400
[a§a§a§a§bbb§c]	10!/7!	6!/(4!×1!×1!^1)	1/1!	21,600
[aaaa§bbb§c]	10!/7!	3!/(1!×1!×1!^1)	1/1!	4,320
[aaaa§b§b§c§c]	10!/7!	5!/(1!×2!^2)	1/2!	10,800
[a§a§a§a§bb§c§c]	10!/7!	7!/(4!×1!×2!)	1/1!	75,600
[a§a§a§a§bb§cc]	10!/7!	6!/(1!×1!^2)	1/2!	259,200
[aaaa§bb§cc]	10!/7!	3!/(1!×1!^2)	1/2!	2,160
[aaa§b§b§b§c§c]	10!/7!	6!/(1! ×3! ×2!)	1/0!	43,200
[aaa§bbb§c§c]	10!/7!	4!/(1!^2 ×2!)	1/2!	4,320
[a§a§a§b§b§b§cc]	10!/7!	7!/(3!^2 ×1!)	1/2!	50,400
[aaa§bbb§cc]	10!/7!	3!/(1!^2×1!)	1/2!	2,160

Table 89 Capacities of eight-digit patterns containing occurrences of exactly four letters, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaaaa§b§c§d]	10!/6!	4!/(1!×1!^3)	1/3!	20,160
[aaaa§b§b§c§d]	10!/6!	5!/(1!×2!×1!^2)	1/2!	151,200
[a§a§a§a§bb§c§d]	10!/6!	7!/(4!×1!×1!^2)	1/2!	529,200
[aaaa§bb§c§d]	10!/6!	4!/(1!×1!×1!^2)	1/2!	60,480
[aaa§b§b§b§c§d]	10!/6!	5!/(1!×3!×1!^2)	1/2!	50,400
[aaa§bbb§c§d]	10!/6!	4!/(1!^2×1!^2)	1/(2!×2!)	30,240
[aaa§b§b§c§c§d]	10!/6!	6!/(1!×2!^2×1!)	1/2!	453,600
[a§a§a§bb§c§c§d]	10!/6!	7!/(3!×2!×1!×1!)	1/1!	2,116,800
[aaa§bb§c§c§d]	10!/6!	5!/(1!×1!×2!×1!)	1/1!	302,400
[a§a§a§bb§cc§d]	10!/6!	6!/(3!×1!^2×1!)	1/2!	302,400
[aaa§bb§cc§d]	10!/6!	4!/(1!×1!^2×1!)	1/2!	60,480
[aa§b§b§c§c§d§d]	10!/6!	7!/(1!×2!^3)	1/3!	529,200
[aa§bb§c§c§d§d]	10!/6!	6!/(1!^2×2!^2)	1/(2!×2!)	226,800
[aa§bb§cc§d§d]	10!/6!	5!/(1!^3×2!)	1/3!	50,400
[aa§bb§cc§dd]	10!/6!	4!/(1!^4)	1/4!	5,040

Table 90 Capacities of eight-digit patterns containing occurrences of exactly five letters, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaaa§b§c§d§e]	10!/5!	5!/(1!×1!^4)	1/4!	151,200
[aaa§b§b§c§d§e]	10!/5!	6!/(1!×2!×1!^3)	1/3!	1,814,400
[a§a§a§bb§c§d§e]	10!/5!	7!/(3!×1!×1!^3)	1/3!	4,233,600
[aaa§bb§c§d§e]	10!/5!	5!/(1!×1!×1!^3)	1/3!	604,800
[aa§b§b§c§c§d§e]	10!/5!	7!/ (1!×2!^2×1!^2)	1/(2!×2!)	9,525,600
[aa§bb§c§c§d§e]	10!/5!	6!/ (1!^2× 2!×1!^2)	1/(2!×2!)	2,721,600
[aa§bb§cc§d§e]	10!/5!	5!/(1!^3×1!^2)	1/(3!×2!)	302,400

Table 91 Capacities of eight-digit patterns containing occurrences of exactly six letters, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aaa§b§c§d§e§f]	10!/4!	6!/(1!×1!^5)	1/5!	907,200
[aa§b§b§c§d§e§f]	10!/4!	7!/(1!× 2!×1!^4)	1/4!	15,876,000
[aa§bb§c§d§e§f]	10!/4!	7!/(1!^2×1!^4)	1/(2!×4!)	15,876,000

Table 92 Capacities of eight-digit patterns containing occurrences of exactly seven letters, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[aa§b§c§d§e§f§g]	10!/3!	7!/(1!×1!^6)	1/6!	4,233,600

Table 93 Capacities of eight-digit patterns containing occurrences of exactly eight letters, with repetitions

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
[a§b§c§d§e§f§g§h]	10!/2!	8!/(1!^8)	1/8!	1,814,400

4.4.6 Bands for patterns representing each number in the complete eight-digit numbering space once only and separating patterns with attractive features to avoid overlaps

Table 94 Bands for eight-digit patterns with clusters and sequences separated out

Band	Pattern	Whether formed (wholly or partly) from sequences	Example	Capacity
A	[a§a§a§a§a§a§a§a]	No	22222222	10
B	[aaaa§bbbb]	Yes	22223333	18
	[aaab§aaab]	Yes	22232223	18
	[aabb§aabb]	Yes	22332233	18
	[ab§ab§ab§ab]	Yes	23232323	18
C	[aa§aa§bb§bb]\[aabb§aabb]	Yes	22333322	36
	[aabc§aabc]	Yes	22342234	48
D	[aa§aa§aa§bb]	Yes	22332222	72
	[aaaa§bbbb]	No	22227777	72
	[aaab§aaab]	No	22272227	72
	[aabb§aabb]	No	22772277	72
	[ab§ab§ab§ab]	No	27272727	72
E	[[a§a§b§b]§[a§a§b§b]]\ [aa§aa§bb§bb]\ [[a§b]§[a§b]§[a§b]§[a§b]]	Yes	23323322	126
	[[a§b]§[a§b]§[a§b]§[a§b]]\ [ab§ab§ab§ab]	Yes	23323223	126
	[[aa§[b§c]]§[aa§[b§c]]]\[aabc§aabc]	Yes	22342243	144
	[a§a§a§a§a§a§a§b]	Yes	22322222	144
	[a§a§a§a§a§a§b§b]\ [[a§a§a§b]§[a§a§a§b]]\ [aa§aa§aa§bb]	Yes	23322222	144
	[aa§aa§bb§bb]\[aabb§aabb]	No	22777722	144
	[abab§cdcd]	Yes	23234545	168
	[abcd§abcd]	Yes	24352435	168
F	[[a§a§a§b]§[a§a§a§b]]\[aaab§aaab]	Yes	22322322	270
	[a§a§a§a§b§b§b§b]\ [[a§a§b§b]§[a§a§b§b]]\ [aaaa§bbbb]	Yes	22223223	288
	[aa§aa§aa§bb]	No	22772222	288
	[aa§aa§[b§c]§[b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	Yes	22223443	384
	[[[a§b]§[a§b]]§[[c§d]§[c§d]]]\ [abab§cdcd]	Yes	23234554	504
	[[a§a§b§b]§[a§a§b§b]]\ [aa§aa§bb§bb]\ [[a§b]§[a§b]§[a§b]§[a§b]]	No	27727722	504

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	[[a§b]§[a§b]§[a§b]§[a§b]]\ [ab§ab§ab§ab]	No	27727227	504
	[a§a§a§a§a§a§b]	No	22722222	576
	[a§a§a§a§a§b§b]\ [[a§a§a§b]§[a§a§a§b]]\ [aa§aa§aa§bb]	No	27722222	576
	[aabc§aabc]	No	22742274	672
	[[a§a§b§b]§[c§c§d§d]]\ [[[a§b]§[a§b]]§[[c§d]§[c§d]]]	Yes	22334455	840
	[a§a§a§a§b§b§b]	Yes	22322323	1,008
	[[a§a§a§b]§[a§a§a§b]]\[aaab§aaab]	No	22722722	1,080
	[a§a§a§b§b§b§b]\ [[a§a§b§b]§[a§a§b§b]]\ [aabb§aabb]	No	22227227	1,152
	[[[a§b]§[c§d]]§[[a§b]§[c§d]]]\ [abcd§abcd]	Yes	23452354	1,176
	[a§a§a§a§a§b§c]	Yes	22322422	1,344
G	[[aa§[b§c]]§[aa§[b§c]]]\[aabc§aabc]	No	22742247	2,016
	[aaa§aaa§b§c]\[aaaaaa§b§c]	No	22272224	2,160
	[aaaaaa§b§c]	No	22222274	2,160
	[[a§b§c§d]§[a§b§c§d]]\ [[[[a§b]§[c§d]]§[[a§b]§[c§d]]]	Yes	23452435	2,688
	[[a§a§b§c]§[a§a§b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	Yes	23242324	3,264
	[a§a§a§a§b§b§b]	No	22722727	4,032
	[abab§cdcd]	No	27274949	4,872
	[abcd§abcd]	No	24792479	4,872
	[aa§aa§aa§b§c]\[aaaaaa§b§c]	No	22722422	5,040
	[aa§aa§[b§c]§[b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	No	22227447	5,376
	[aab§aab§c§d]	Yes	22342235	6,048
	[a§a§a§b§b§c§c]\ [[a§a§b§c]§[a§a§b§c]]\ [aa§aa§[b§c]§[b§c]]	Yes	23223424	6,240
	[a§a§a§a§b§b§c]	Yes	22322324	8,064
	[a§a§a§a§b§c§d]	Yes	22322524	9,408
	[a§a§a§a§a§b§c]\ [aaa§aaa§b§c]\ [aa§aa§aa§b§c]	No	22722242	9,456
	[ab§ab§ab§c§d]	Yes	23235423	10,080
	[a§a§b§b§c§c§d§d]\ [[a§b§c§d]§[a§b§c§d]\ [a§a§b§b]§[c§c§d§d]]	Yes	22343455	12,096
	[abc§abc§d§e]	Yes	23452346	12,096
	[a§a§a§a§b§b§b§c]	Yes	22322343	13,440
	[a§a§a§b§b§b§c§c]	Yes	22332344	13,440

	[[[a§b]§[a§b]]§[[c§d]§[c§d]]]\ [abab§cdcd]	No	27274994	14,616
H	[aab§aab§c§d]	No	22742279	24,192
	[[a§a§b§b]§[c§c§d§d]]\ [[[a§b]§[a§b]]§[[c§d]§[c§d]]]	No	22774499	24,360
	[[a§b]§[a§b]§[a§b]§c§d]\ [ab§ab§ab§c§d]	Yes	23423523	30,240
	[[[a§b]§[c§d]]§[[a§b]§[c§d]]]\ [abcd§abcd]	No	27492794	34,104
	[ab§ab§ab§c§d]	No	27279427	40,320
	[[a§a§b§c]§[a§a§b§c]]\ [[aa§[b§c]]§[aa§[b§c]]]	No	27242724	45,696
	[[a§a§b]§[a§a§b]§c§d]\ [aab§aab§c§d]	Yes	23242235	48,384
	[a§a§a§a§b§c§d§e]	Yes	22322456	50,400
	[[a§b§c]§[a§b§c]§d§e]\ [abc§abc§d§e]	Yes	24352346	60,480
	[[a§b§c§d]§[a§b§c§d]]\ [[[[a§b]§[c§d]]§[[a§b]§[c§d]]]	No	27492479	77,952
	[a§a§a§a§b§b§c§c]\ [[a§a§b§c]§[a§a§b§c]]\ [aa§aa§[b§c]§[b§c]]	No	27227424	87,360
	[a§a§a§a§a§b§b§c]	No	22722724	112,896
	[[a§b]§[a§b]§[a§b]§c§d]\ [ab§ab§ab§c§d]	No	27427927	120,960
	[a§b§c§d§e§f§g§h]	Yes	24359786	120,960
	[a§a§a§b§b§c§c§d]	Yes	22323445	141,120
	[abc§abc§d§e]	No	27492746	169,344
	[a§a§a§a§b§b§b§c]	No	22722747	188,160
	[a§a§a§b§b§b§c§c]	No	22772744	188,160
	[[a§a§b]§[a§a§b]§c§d]\ [aab§aab§c§d]	No	27242279	193,536
	[a§a§a§b§c§d§e§f]	Yes	24352726	201,600
	[a§a§a§b§b§b§c§d]\ [[a§b]§[a§b]§[a§b]§c§d]	Yes	22323345	241,920
	[a§a§a§a§a§b§c§d]	No	22722924	272,832
	[a§a§b§b§c§c§d§d]\ [[a§b§c§d]§[a§b§c§d]\ [a§a§b§b]§[c§c§d§d]]	No	22747499	350,784
	[a§a§a§a§b§b§c§d]\ [[a§a§b]§[a§a§b]§c§d]	Yes	22324325	368,928
	[a§a§a§b§b§c§d§e]	Yes	22324356	403,200
	[[a§b]§[a§b]§c§d§e§f]\ [ab§ab§c§d§e§f]	Yes	23432756	453,600
	[aa§aa§b§c§d§e]	No	22722496	453,600
	[ab§ab§c§d§e§f]	Yes	23452376	453,600
	[a§a§b§c§d§e§f§g]	Yes	24352786	564,480

	[a§a§b§b§c§c§d§e]\ [[a§b§c]§[a§b§c]§d§e]	Yes	24345236	774,144
	[[a§b§c]§[a§b§c]§d§e]\ [abc§abc§d§e]	No	24792746	846,720
	[a§a§a§b§b§b§c§d]\ [[a§b]§[a§b]§[a§b]§c§d]	No	22727749	967,680
I	[a§a§a§a§b§b§c§d]\ [[a§a§b]§[a§a§b]§c§d]	No	22724729	1,475,712
	[a§a§a§a§b§c§d§e]\ [aa§aa§b§c§d§e]	No	22724926	1,612,800
	[a§b§c§d§e§f§g§h]	No	24793586	1,693,440
	[[a§b]§[a§b]§c§d§e§f]\ [ab§ab§c§d§e§f]	No	27472596	1,814,400
	[ab§ab§c§d§e§f]	No	27492756	1,814,400
J	[a§a§a§b§b§c§c§d]	No	22727449	4,092,480
	[a§a§b§b§c§d§e§f]\ [[a§b]§[a§b]§c§d§e§f]	Yes	23452736	5,443,200
K	[a§a§a§b§c§d§e§f]	No	24792526	8,265,600
	[a§a§b§b§c§c§d§e]\ [[a§b§c]§[a§b§c]§d§e]	No	24749276	10,838,016
L	[a§a§b§c§d§e§f§g]	No	24792586	16,369,920
	[a§a§a§b§b§c§d§e]	No	22724796	16,531,200
	[a§a§b§b§c§d§e§f]\ [[a§b]§[a§b]§c§d§e§f]	No	27492576	21,772,800

5 Enumerating patterns

This section presents techniques that are built on part of the theory in Section 4 but that can be used instead of extensions to that theory. It applies the techniques to calculate the capacities of further patterns and develop different ways of classifying and banding numbers. Among the examples treated fully are both the complete eight-digit numbering space and a restricted eight-digit numbering space in which '0' is never the second digit of a number. The techniques can be applied without a full appreciation of the theory, or even the notation, of Section 4.

5.1 Capacity calculation

5.1.1 Descriptors of patterns

In the classification of numbers summarised in Section 4.2.4, "small" patterns that have attractive features are composed using jumbles and other operations on sets. That classification has the limitations documented in Section 4.2.5. Here, instead, "large" patterns are decomposed to separate out those that have attractive features. The emphasis is not on composing patterns from letters but on extracting information about attractive features from the properties of patterns.

Perhaps the most fundamental properties of a pattern are its quantity of distinct digits and and the quantities of individual digits. One of these can be derived from the others, as the quantities of individual digits collectively sum to the length of the pattern. For this reason one further definition is useful.

A 'descriptor' of a pattern is a numeric character string that specifies the quantities of each distinct digit in the pattern. For instance:

- '22222222' has '8' as its descriptor (because in it there are eight occurrences of '2').
- '22222227' has '71' as its descriptor (because in it there are seven occurrences of '2' and one occurrence of '7').
- '22222277' has '62' as its descriptor (because in it there are six occurrences of '2' and two occurrences of '7').
- '22222247' has '611' as its descriptor (because in it there are six occurrences of '2', one occurrence of '4' and one occurrence of '7').
- '22222447', '22224427' and '24222427' all have '521' as their descriptor (because in each there are five occurrences of '2', two occurrences of '4' and one occurrence of '7').

- '24222459', '22522494' and '24252429' all have '4211' as their descriptor (because in each there are four occurrences of '2', two occurrences of '4', one occurrence of '5' and one occurrence of '9').
- '27274459', '27542749' and '27452749' all have '22211' as their descriptor (because in each there are two occurrences of '2', two occurrences of '7' two occurrences of '4', one occurrence of '5' and one occurrence of '9').

A descriptor is intended to be a series of digits, so, for example, '71' is pronounced "seven one", not "seventy-one", and '4211' is pronounced "four two one one", not "four thousand two hundred and eleven".

5.1.2 Capacities for basic patterns

The tables below copy the tables of Section 4.1.4 for the 10,000 four-digit numbers, the 100,000 five-digit numbers, the 1,000,000 six-digit numbers and the 10,000,000 seven-digit numbers, except that they demonstrate the use of descriptors instead of jumbles to identify the basic patterns. They continue to show how the capacities are obtained from the digit choice counts, letter order counts and overlap extents.

Table 95 Capacities of four-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
4	10!/9!	4!/(4!×0!)	1/0!	10
31	10!/8!	4!/(3!×1!)	1/1!	360
22	10!/8!	4!/(2!^2)	1/2!	270
211	10!/7!	4!/(2!×1!^2)	1/2!	4,320
1111	10!/6!	4!/(1!^4)	1/4!	5,040

Table 96 Capacities of five-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
5	10!/9!	5!/(5!×0!)	1/0!	10
41	10!/8!	5!/(4!×1!)	1/1!	450
32	10!/8!	5!/(3!× 2!)	1/1!	900
311	10!/7!	5!/(3!×1!^2)	1/2!	7,200
221	10!/7!	5!/(2!^2×1!^1)	1/2!	10,800
2111	10!/6!	5!/(2!×1!^3)	1/3!	50,400
11111	10!/5!	5!/(1!^5)	1/5!	30,240

Table 97 Capacities of six-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
6	10!/9!	6!/(6!×0!)	1/0!	10
51	10!/8!	6!/(5!×1!)	1/1!	540
42	10!/8!	6!/(4!× 2!)	1/1!	1,350
33	10!/8!	6!/(3!^2)	1/2!	900
411	10!/7!	6!/(4!×1!^2)	1/2!	10,800
321	10!/7!	6!/(3!×2!×1!^1)	1/1!	43,200
222	10!/7!	6!/(2!^3)	1/3!	10,800
3111	10!/6!	6!/(3!×1!^3)	1/3!	100,800
2211	10!/6!	6!/(2!^2×1!^2)	1/(2!×2!)	226,800
21111	10!/5!	6!/(2!×1!^4)	1/4!	453,600
111111	10!/4!	6!/(1!^6)	1/6!	151,200

Table 98 Capacities of seven-digit patterns representing each number uniquely

Pattern	Digit choice count	Letter order count	Overlap extent	Capacity
7	10!/9!	7!/(7!×0!)	1/0!	10
61	10!/8!	7!/(6!×1!)	1/1!	630
52	10!/8!	7!/(5!× 2!)	1/1!	1,890
43	10!/8!	7!/(4!×3!)	1/1!	3,150
511	10!/7!	7!/(5!×1!^2)	1/2!	15,120
421	10!/7!	7!/(4!×2!×1!^1)	1/1!	75,600
331	10!/7!	7!/(3!^2×1!^1)	1/2!	50,400
322	10!/7!	7!/(3!×2!^2)	1/2!	75,600
4111	10!/6!	7!/(4!×1!^3)	1/3!	176,400
3211	10!/6!	7!/(3!×2!×1!^2)	1/2!	1,058,400
2221	10!/6!	7!/(2!^3×1!^1)	1/3!	529,200
31111	10!/5!	7!/(3!×1!^4)	1/4!	1,058,400
22111	10!/5!	7!/(2!^2×1!^3)	1/(2!×3!)	3,175,200
211111	10!/4!	7!/(2!×1!^5)	1/5!	3,175,200
1111111	10!/3!	7!/(1!^7)	1/7!	604,800

5.2 Pattern classification

5.2.1 Patterns in numbers

The 'attractive features' of patterns considered below are as follows:

- Few distinct digits (as characterised by the quantities of distinct digits in the patterns, with the effect that '22222227', '22222277' and '22222247' might be in different classes).
- Repetitions of digits (as characterised by the arrangements of the digits in the repetitions, with the effect that '22222447', '22224427' and '24222427' might be in different classes).
- Clusters of digits (as characterised by the arrangements of the digits in the clusters, with the effect that '27274459', '27542749' and '27452749' might be in different classes).
- Sequences of digits (as characterised by the lengths of the sequences such that digits rise in steps of one, as in '01234', or fall in steps of one, as in '98765').

In an eight-digit numbering space, for any pattern:

- The quantity of distinct digits is between one and eight, and the quantities of individual digits collectively sum to eight. These quantities limit the lengths of the repetitions, clusters and sequences included in the pattern, but they do not state those lengths, unless the pattern takes a particular form; for instance, '22222447', '22224427' and '24222427' all have five occurrences of '2', two occurrences of '4' and one occurrence of '7', but their longest repetitions have five, four and three digits respectively.
- The length of the longest repetition is between two and eight. If it is three (for example) the pattern is said to have a "three-digit repetition".
- The length of the longest cluster is between two and four. If it is four (for example) the pattern is said to have a "four-digit cluster".
- The length of the longest sequence is between two and eight. If it is five (for example) the pattern is said to have a "five-digit sequence". In fact the only sequences considered here are those that have more than four digits rising or falling in steps of one, as they are likely to be the ones most easily recognised and most widely regarded as special.

5.2.2 Examination of patterns in the numbering space

To identify the patterns having attractive features, the numbering space is examined thus:

- The basic patterns that represent all arrangements of digits are provided as in Section 4.4.1. Each such pattern has its own digit choice count, letter order count and overlap extent. In these basic patterns different letters without subscripts represent different digits, so different orderings of the letters represent different arrangements of the digits.
- The different orderings of letters are listed for each basic pattern. They are then tested to determine the repetitions and clusters included in them.
- The quantities of orderings of letters that include longest repetitions with a given length and longest clusters with a given length are counted for each basic pattern. The counts distinguish between different combinations of lengths of longest repetitions and clusters: orderings of letters can have both longest repetitions with a given length and longest clusters with a given length, longest repetitions with a given length but not clusters, longest clusters with a given length but not repetitions, or no repetitions and clusters.
- The different arrangements of digits that include sequences are listed for each length of sequence, as tabulated in Section 5.2.3. They are then assigned to basic patterns and tested to determine the repetitions and clusters included in them.
- The quantities of arrangements of digits that include longest repetitions with a given length, longest clusters with a given length and longest sequences with a given length are counted for each basic pattern. The counts distinguish again between different combinations of lengths of longest repetitions, clusters and sequences.

By these procedures arrangements of digits with different combinations of lengths of longest repetitions, clusters and sequences are separated out from each basic pattern. In effect the numbers in the numbering space are classified according to their basic patterns and their combinations of lengths of longest repetitions, clusters and sequences.

For an eight-digit numbering space there might be repetitions with lengths between two and eight, clusters with lengths between two and four and sequences with lengths between five and eight. As there might not be repetitions, clusters or sequences in arrangements of digits, there might be 8×4×5 combinations of lengths for each basic pattern; however, each of the basic patterns is consistent with only some of them.

As demonstrated in Section 4.4.1, for the complete eight-digit numbering space, containing every eight-digit number, there are twenty-two basic patterns that together represent each eight-digit number exactly once. Their letter order counts range between one and 40,320. For each of them the letter order count is the quantity of orderings of letters that must be listed and tested; in total there are 95,503 such orderings, distributed very unequally among the twenty-two basic patterns. Though this is a large total, it is much smaller than the total of 100,000,000 numbers that would be obtained by multiplying the letter order counts by the digit choice counts and overlap extents. Moreover, there are 178, not 8×4×5×22 (which is 3,520), classes of numbers specified by combinations of lengths of longest repetitions, clusters and sequences consistent with particular basic patterns. The use of basic patterns thus allows the numbering space to be split into parts that can be examined separately.

Each of the basic patterns represents all the numbers that have particular quantities of distinct digits. For the complete eight-digit numbering space, with no restrictions on the positions for the digits, the digits that can occur in the first position (say) are the same as those that can occur in the second. Consequently a basic pattern that represents numbers in which there are four (say) occurrences of the digit that occurs in the first position also represents numbers in which there are four occurrences of the digit that occurs in the second position.

However, for the restricted numbering space. In which '0' never occurs in the second position in numbers, the digits that can occur in the first position are not the same as those that can occur in the second. Consequently a basic pattern that represents numbers in which there are four occurrences of the digit that occurs in the first position does not also represent numbers in which there are four occurrences of the digit that occurs in the second position. The quantity of basic patterns is approximately doubled, to distinguish the numbers in which the digit in the second position is replicated from the numbers in which the digit in the first position (or any other position, except the second) is replicated. In fact there are forty-five basic patterns for the restricted numbering space. Also, there are 365 classes of numbers specified by combinations of lengths of longest repetitions, clusters and sequences consistent with particular ones of these basic patterns.

5.2.3 Capacities for sequences

By contrast with sequences defined as in Section 4.2.1, sequences defined as in Section 5.2.1 do not provide simple distinctions between a sequence and the remainder of the pattern in which it appears. Here the rules governing when a sequence can appear in a pattern depend on the position of the sequence in the pattern and the selection of digits in the sequence, not just on the quantity of digits in the sequence and the remainder of the pattern. For instance, among five-digit sequences (excluding any that are really parts of six-digit sequences):

- There are 900 represented by '01234***', 900 represented by '*01234**', 900 represented by 'ab01234c' and 1,000 represented by '***01234'.
- There are 900 represented by '12345***', 810 represented by '*12345**', 810 represented by '**12345*' and 900 represented by '***12345'.

Consequently here the procedure for determining which patterns include sequences is different from the procedure for determining which patterns include repetitions or clusters: it entails examining arrangements of digits, instead of orderings of letters.

For the complete eight-digit numbering space the quantities of sequences having eight, seven, six and five digits range between three and 20,180. In total there are 22,500 such sequences, distributed among the four lengths of sequence in proportions that rise by over a factor of ten for each unit decrease in length. These figures are doubled if both ascending and descending sequences are admitted (as they are in this report).

Table 99 shows the quantities of these sequences, calculated just from the position and selection of the first digit of each sequence. For each row of the table the digit choice count is the quantity of distinct combinations of digits that can contain the sequences identified in the row.

The table omits sequences having fewer than five digits not merely because they are not so easily recognised but also because the calculations become more complicated to display: in an eight-digit number a four-digit sequence, for example, might be followed by another four-digit sequence, but only one of them should add to the total quantity of sequences. Also, numbers that themselves have fewer than five distinct digits are likely to be regarded as attractive, regardless of whether they contain digits ascending or descending in steps of one.

Table 99 Quantities of eight-digit numbers having sequences with more than four ascending or descending digits in the complete eight-digit numbering space

Length of sequence	Ascending sequence first digit		Descending sequence first digit		Digit choice count	Quantity
	Position	Selection	Position	Selection		
8	1	'0', '1', '2'	1	'9', '8', '7'	3+3	6
7	1	'0', '1', '2'	2	'8', '7', '6'	3×9+9×3	54
7	1	'3'	2	'9'	1×10+10×1	20
7	2	'0'	1	'6'	10×1+1×10	20
7	2	'1', '2', '3'	1	'9', '8', '7'	9×3+3×9	54
6	1	'0', '1', '2', '3'	3	'8', '7', '6', '5'	4×9×10+10×9×4	720
6	1	'4'	3	'9'	1×10×10+10×10×1	200
6	2	'0'	2	'5'	10×1×9+9×1×10	180
6	2	'1', '2', '3'	2	'8', '7', '6'	9×3×9+9×3×9	486
6	2	'4'	2	'9'	9×1×10+10×1×9	180

6	3	‘0’	1	‘5’	10×10×1+1×10×10	200
6	3	‘1’, ‘2’, ‘3’, ‘4’	1	‘9’, 8’, ‘7’, ‘6’	10×9×4+4×9×10	720
5	1	‘0’, ‘1’, ‘2’, ‘3’, ‘4’	4	‘8’, ‘7’, ‘6’, ‘5’, ‘4’	5×9×10×10+10×10×9×5	9,000
5	1	‘5’	4	‘9’	1×10×10×10+10×10×10×1	2,000
5	2	‘0’	3	‘4’	10×1×9×10+10×9×1×10	1,800
5	2	‘1’, ‘2’, ‘3’, ‘4’	3	‘8’, ‘7’, ‘6’, ‘5’	9×4×9×10+10×9×4×9	6,480
5	2	‘5’	3	‘9’	9×1×10×10+10×10×1×9	1,800
5	3	‘0’	2	‘4’	10×10×1×9+9×1×10×10	1,800
5	3	‘1’, ‘2’, ‘3’, ‘4’	2	‘8’, ‘7’, ‘6’, ‘5’	10×9×4×9+9×4×9×10	6,480
5	3	‘5’	2	‘9’	10×9×1×10+10×1×9×10	1,800
5	4	‘0’	1	‘4’	10×10×10×1+1×10×10×10	2,000
5	4	‘1’, ‘2’, ‘3’, ‘4’, ‘5’	1	‘9’, ‘8’, ‘7’, ‘6’, ‘5’	10×10×9×5+5×9×10×10	9,000

In this report both ascending and descending sequences are allowed to be special. For the complete eight-digit numbering space, there is symmetry in the treatment of ascending and descending sequences, so the quantities of sequences are simply doubled, as in the table. However, this symmetry is broken in the restricted eight-digit numbering space in which ‘0’ never occurs in the second position in numbers: ascending sequences starting with ‘0’ must not start in the second position, but descending sequences starting with ‘9’ are under no such constraint. Table 100 shows the effect of breaking the symmetry; altogether 2,050 ascending sequences and 1,150 descending sequences are invalid in the restricted numbering space.

Table 100 Quantities of eight-digit numbers having sequences with more than four ascending or descending digits in the restricted eight-digit numbering space

Length of sequence	Ascending sequence first digit		Descending sequence first digit		Digit choice count	Quantity
	Position	Selection	Position	Selection		
8	1	‘0’, ‘1’, ‘2’	1	‘9’, ‘8’, ‘7’	3+3	6
7	1	‘0’, ‘1’, ‘2’	2	‘8’, ‘7’, ‘6’	3×9+9×3	54
7	1	‘3’	2	‘9’	1×10+10×1	20
7	2		1	‘6’	10×0+1×10	10
7	2	‘1’, ‘2’, ‘3’	1	‘9’, ‘8’, ‘7’	9×3+3×9	54
6	1	‘0’, ‘1’, ‘2’, ‘3’	3	‘8’, ‘7’, ‘6’, ‘5’	4×9×10+10×8×4	680
6	1	‘4’	3	‘9’	1×10×10+10×9×1	190
6	2		2	‘5’	10×0×9+9×1×10	90
6	2	‘1’, ‘2’, ‘3’	2	‘8’, ‘7’, ‘6’	9×3×9+9×3×9	486
6	2	‘4’	2	‘9’	9×1×10+10×1×9	180
6	3	‘0’	1	‘5’	10×9×1+1×10×10	190
6	3	‘1’	1	‘6’	10×9×1+1×9×10	180
6	3	‘2’, ‘3’, ‘4’	1	‘9’, 8’, ‘7’	10×8×3+3×9×10	510

5	1	‘0’, ‘1’, ‘2’, ‘3’, ‘4’	4	‘8’, ‘7’, ‘6’, ‘5’, ‘4’	5×9×10×10+10×9×9×5	8,550
5	1	‘5’	4	‘9’	1×10×10×10+10×9×10×1	1,900
5	2		3	‘4’	10×0×9×10+10×8×1×10	800
5	2	‘1’, ‘2’, ‘3’, ‘4’	3	‘8’, ‘7’, ‘6’, ‘5’	9×4×9×10+10×8×4×9	6,120
5	2	‘5’	3	‘9’	9×1×10×10+10×9×1×9	1,710
5	3	‘0’	2	‘4’	10×9×1×9+9×1×10×10	1,710
5	3	‘1’	2	‘5’	10×9×1×9+9×1×9×10	1,620
5	3	‘2’, ‘3’, ‘4’	2	‘8’, ‘7’, ‘6’	10×8×3×9+9×3×9×10	4,590
5	3	‘5’	2	‘9’	10×8×1×10+10×1×9×10	1,700
5	4	‘0’	1	‘4’	10×9×10×1+1×10×10×10	1,900
5	4	‘1’, ‘2’, ‘3’, ‘4’, ‘5’	1	‘9’, ‘8’, ‘7’, ‘6’, ‘5’	10×9×9×5+5×9×10×10	8,550

5.2.4 Tables for the classification

The 178 combinations of lengths of longest repetitions, clusters and sequences consistent with particular basic patterns for the complete eight-digit numbering space are tabulated in Section 5.4.1. Each of the tables displays together, for convenience, all of combinations for all of the basic patterns having a particular quantity of distinct digits; however, each of the twenty-two basic patterns is listed and tested on its own.

Because the basic patterns represent each number exactly once, for every arrangement of digits there is a ‘related basic pattern’, which is the basic pattern that represents the same digits (in that arrangement, among others) and that has the same restrictions on what those digits may be. Each of the entries in any of these tables gives the lengths of the longest repetition, cluster and sequence in a particular arrangement of digits, and the related basic pattern. The lengths are necessarily no more than the lengths of the longest repetition, cluster and sequence in the related basic pattern, which are identified in Section 5.2.5.

Different related basic patterns and different lengths of the longest repetitions, clusters and sequences have different entries in the tables of Section 5.4.1. In fact in the tables a pattern is specified by its related basic pattern and the lengths of its longest repetition, cluster and sequence; the column headings for the related basic pattern and the lengths of the longest repetition, cluster and sequence are written in black, instead of white, to indicate that they specify a pattern instead of being derived from a pattern (by contrast with the column headings in Section 5.2.6).

The tables of Section 5.4.1 are merged and sorted in Section 5.4.2, so that the patterns can be put into bands (A, B, C, D and E). As explained in Section 5.3.1, the bands provide three ways of examining ever more of the attractive features of patterns determined by the descriptors (“D”), repetitions (“R”), clusters (“C”) and sequences (“S”).

There is a similar analysis of the restricted eight-digit numbering space in which '0' is never the second digit of a number. This restriction makes the numbering space roughly twice as complicated as the complete eight-digit numbering space, even though it has only 90,000,000 numbers: while the complete numbering space has twenty-two basic patterns and is split into 178 classes of numbers, the restricted eight-digit numbering space has forty-five basic patterns and is split into 365 classes of numbers. These classes are tabulated in Section 5.4.4. Again the tables are merged and sorted into bands in Section 5.4.5, using the technique for defining bands given in Section 5.3.1.

5.2.5 Inferences from descriptors

As illustrated in Section 5.1.2, a basic pattern is identified uniquely by a descriptor, just as it is identified uniquely by a jumble. The descriptor encodes useful information, in that:

- The length of the descriptor is the total quantity of distinct digits in each number represented by the basic pattern.
- The highest digit in the descriptor is the length of the longest repetition in the numbers represented by the basic pattern (provided that this digit is at least two).
- The sum of half (rounded downwards) of each digit in the descriptor is the length of the longest cluster in the numbers represented by the basic pattern (provided that this sum, and two digits in the descriptor, are at least two).
- The length of the descriptor is the length of the longest sequence in the numbers represented by the basic pattern (provided that this length is at least five, if sequences are required to have more than four digits).

These quantities indicate the most attractive features of numbers represented by the basic pattern. Usually many numbers represented by the basic pattern do not have these features: their repetitions, clusters and sequences can be shorter than the quantities allow. Nonetheless the quantities are relevant to all of the numbers represented by the basic pattern because they indicate the "best possible" features.

A pattern other than a basic pattern is not identified uniquely by a descriptor; because one pattern might have the same quantities of each digits as another. However, for any pattern there is a basic pattern to which it is related that represents the same digits and has the same restrictions on what those digits may be. As the pattern has the same descriptor as its related basic pattern:

- The length of the longest repetition in the pattern is no more than the highest digit in the descriptor. For there to be any such repetition, this quantity must be at least two. Accordingly in patterns with descriptors '8', '71', '62', '611', '521', '4211', '32111', '22211' and '221111' there might be repetitions with lengths eight, seven, six, six, five, four, three, two and two respectively.

- The length of the longest cluster in the pattern is no more than the sum of half (rounded downwards) of each digit in the descriptor. For there to be any such cluster, this sum, and two digits in the descriptor, must be at least two. Accordingly in patterns with descriptors '8', '71' and '611' there are no clusters, and in patterns with descriptors '62', '521', '4211', '32111', '22211' and '221111' there might be clusters with lengths four, three, three, two, three and two respectively.
- The length of the longest sequence in the pattern is no more than the length of the descriptor. For there to be any such sequence, this length must be at least five (if sequences are required to have more than four ascending or descending digits). Accordingly in patterns with descriptors '8', '71', '62', '611', '521' and '4211' there are no sequences, and in patterns with descriptors '32111', '22211' and '221111' there might be sequences with length five, five and six respectively.

5.2.6 Properties of basic patterns

The calculation of capacities for basic patterns is illustrated in Section 4.1.4 (where the basic patterns are identified using jumbles) and Section 5.1.2 (where the basic patterns are identified using descriptors). Other useful quantities can also be calculated without examining the basic patterns in detail. In particular, the lengths of the longest repetitions, clusters and sequences for the basic patterns can be derived from the jumbles or the descriptors (as specified in Section 5.2.5).

Table 101 exhibits the lengths of the longest repetitions, clusters and sequences for the twenty-two basic patterns needed by the 100,000,000 numbers of the complete eight-digit numbering space.

By limiting the lengths of the repetitions, clusters and sequences in the numbers that they represent, the basic patterns can be useful in assessing which numbers are likely to have attractive features. However, more exact assessments are needed, at least when patterns have several distinct digits. These are provided by the classification summarised in Section 5.2.4: numbers are placed in classes specified by their related basic patterns and by the lengths of their own longest repetitions, clusters and sequences, even where other numbers represented by the related basic patterns have longer repetitions, clusters and sequences. These classes are the ones adopted for the tabulation of the complete eight-digit numbering space in Section 5.4.1. There is then an important distinction between the lengths of the longest repetitions, clusters and sequences tabulated here and those tabulated in Section 5.4.1: those tabulated here can be derived from the descriptors of the basic patterns, while those tabulated in Section 5.4.1 work alongside the descriptors of the basic patterns to specify classes of numbers.

Table 101 Properties of basic patterns in the complete eight-digit numbering space

Pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	Digit choice count	Letter order count	Overlap extent	Capacity
8	8			10!/ (10-1)!	8!/ (8!^1)	1/ 1!	10
71	7			10!/ (10-2)!	8!/ (1!^1×7!^1)	1/ (1!×1!)	720
62	6	4		10!/ (10-2)!	8!/ (2!^1×6!^1)	1/ (1!×1!)	2,520
53	5	3		10!/ (10-2)!	8!/ (3!^1×5!^1)	1/ (1!×1!)	5,040
44	4	4		10!/ (10-2)!	8!/ (4!^2)	1/ 2!	3,150
611	6			10!/ (10-3)!	8!/ (1!^2×6!^1)	1/ (2!×1!)	20,160
521	5	3		10!/ (10-3)!	8!/ (1!^1×2!^1×5!^1)	1/ (1!×1!×1!)	120,960
431	4	3		10!/ (10-3)!	8!/ (1!^1×3!^1×4!^1)	1/ (1!×1!×1!)	201,600
422	4	4		10!/ (10-3)!	8!/ (2!^2×4!^1)	1/ (2!×1!)	151,200
332	3	3		10!/ (10-3)!	8!/ (2!^1×3!^2)	1/ (1!×2!)	201,600
5111	5			10!/ (10-4)!	8!/ (1!^3×5!^1)	1/ (3!×1!)	282,240
4211	4	3		10!/ (10-4)!	8!/ (1!^2×2!^1×4!^1)	1/ (2!×1!×1!)	2,116,800
3311	3	2		10!/ (10-4)!	8!/ (1!^2×3!^2)	1/ (2!×2!)	1,411,200
3221	3	3		10!/ (10-4)!	8!/ (1!^1×2!^2×3!^1)	1/ (1!×2!×1!)	4,233,600
2222	2	4		10!/ (10-4)!	8!/ (2!^4)	1/ 4!	529,200
41111	4		5	10!/ (10-5)!	8!/ (1!^4×4!^1)	1/ (4!×1!)	2,116,800
32111	3	2	5	10!/ (10-5)!	8!/ (1!^3×2!^1×3!^1)	1/ (3!×1!×1!)	16,934,400
22211	2	3	5	10!/ (10-5)!	8!/ (1!^2×2!^3)	1/ (2!×3!)	12,700,800
311111	3		6	10!/ (10-6)!	8!/ (1!^5×3!^1)	1/ (5!×1!)	8,467,200
221111	2	2	6	10!/ (10-6)!	8!/ (1!^4×2!^2)	1/ (4!×2!)	31,752,000
2111111	2		7	10!/ (10-7)!	8!/ (1!^6×2!^1)	1/ (6!×1!)	16,934,400
11111111			8	10!/ (10-8)!	8!/ (1!^8)	1/ 8!	1,814,400

Another example is provided by the restricted eight-digit numbering space in which '0' is never the second digit of a number. For this case in this report a descriptor has as its first digit the quantity of occurrences of the digit in the second position of a number (though the choice of the first digit in the descriptor is just to be definite, as any digit would do). That quantity of occurrences is not necessarily the highest in the number, so the first digit of the descriptor is not necessarily the highest in the descriptor. However, positions in numbers other than the second are essentially equivalent, so the remaining digits in the descriptor (or the remaining letters in the jumble for the basic pattern) can be ordered in any way; ordering them from the highest to the lowest is convenient but not significant, bearing in mind, for example, that the descriptor '1241' can be replaced by '1421' (though not by '4211' or '2411' because of the convention about how the descriptors begin). There are in fact forty-five descriptors of basic patterns for the restricted numbering space.

Because the digit in the second position is not '0', the digit choice count allows only nine choices for that digit, the letter order count allows only seven positions for the other digits, and the overlap extent allows only those other digits to be treated as having the same constraints and as occurring the same number of times in analogous positions. Table 102 demonstrates the resulting calculations.

Table 102 Properties of basic patterns in the restricted eight-digit numbering space

Pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	Digit choice count	Letter order count	Overlap extent	Capacity
8	8			9×9!/ (9-(1-1))!	7!/ ((8-1)!)	1/ 0!	9
71	7			9×9!/ (9-(2-1))!	7!/ ((7-1)!×1!^1)	1/ 1!	567
62	6	4		9×9!/ (9-(2-1))!	7!/ ((6-1)!×2!^1)	1/ 1!	1,701
53	5	3		9×9!/ (9-(2-1))!	7!/ ((5-1)!×3!^1)	1/ 1!	2,835
44	4	4		9×9!/ (9-(2-1))!	7!/ ((4-1)!×4!^1)	1/ 1!	2,835
35	5	3		9×9!/ (9-(2-1))!	7!/ ((3-1)!×5!^1)	1/ 1!	1,701
26	6	4		9×9!/ (9-(2-1))!	7!/ ((2-1)!×6!^1)	1/ 1!	567
17	7			9×9!/ (9-(2-1))!	7!/ ((1-1)!×7!^1)	1/ 1!	81
611	6			9×9!/ (9-(3-1))!	7!/ ((6-1)!×1!^2)	1/ 2!	13,608
521	5	3		9×9!/ (9-(3-1))!	7!/ ((5-1)!×1!^1×2!^1)	1/ (1!×1!)	68,040
431	4	3		9×9!/ (9-(3-1))!	7!/ ((4-1)!×1!^1×3!^1)	1/ (1!×1!)	90,720

422	4	4		9×9!/ (9-(3-1))!	7!/ ((4-1)!×2!^2)	1/ 2!	68,040
341	4	3		9×9!/ (9-(3-1))!	7!/ ((3-1)!×1!^1×4!^1)	1/ (1!×1!)	68,040
332	3	3		9×9!/ (9-(3-1))!	7!/ ((3-1)!×2!^1×3!^1)	1/ (1!×1!)	136,080
251	5	3		9×9!/ (9-(3-1))!	7!/ ((2-1)!×1!^1×5!^1)	1/ (1!×1!)	27,216
242	4	4		9×9!/ (9-(3-1))!	7!/ ((2-1)!×2!^1×4!^1)	1/ (1!×1!)	68,040
233	3	3		9×9!/ (9-(3-1))!	7!/ ((2-1)!×3!^2)	1/ 2!	45,360
161	6			9×9!/ (9-(3-1))!	7!/ ((1-1)!×1!^1×6!^1)	1/ (1!×1!)	4,536
152	5	3		9×9!/ (9-(3-1))!	7!/ ((1-1)!×2!^1×5!^1)	1/ (1!×1!)	13,608
143	4	3		9×9!/ (9-(3-1))!	7!/ ((1-1)!×3!^1×4!^1)	1/ (1!×1!)	22,680
5111	5			9×9!/ (9-(4-1))!	7!/ ((5-1)!×1!^3)	1/ 3!	158,760
4211	4	3		9×9!/ (9-(4-1))!	7!/ ((4-1)!×1!^2×2!^1)	1/ (2!×1!)	952,560
3311	3	2		9×9!/ (9-(4-1))!	7!/ ((3-1)!×1!^2×3!^1)	1/ (2!×1!)	952,560
3221	3	3		9×9!/ (9-(4-1))!	7!/ ((3-1)!×1!^1×2!^2)	1/ (1!×2!)	1,428,840
2411	4	3		9×9!/ (9-(4-1))!	7!/ ((2-1)!×1!^2×4!^1)	1/ (2!×1!)	476,280
2321	3	3		9×9!/ (9-(4-1))!	7!/ ((2-1)!×1!^1×2!^1×3!^1)	1/ (1!×1!×1!)	1,905,120
2222	2	4		9×9!/ (9-(4-1))!	7!/ ((2-1)!×2!^3)	1/ 3!	476,280
1511	5			9×9!/ (9-(4-1))!	7!/ ((1-1)!×1!^2×5!^1)	1/ (2!×1!)	95,256
1421	4	3		9×9!/ (9-(4-1))!	7!/ ((1-1)!×1!^1×2!^1×4!^1)	1/ (1!×1!×1!)	476,280
1331	3	2		9×9!/ (9-(4-1))!	7!/ ((1-1)!×1!^1×3!^2)	1/ (1!×2!)	317,520
1322	3	3		9×9!/ (9-(4-1))!	7!/ ((1-1)!×2!^2×3!^1)	1/ (2!×1!)	476,280
41111	4		5	9×9!/ (9-(5-1))!	7!/ ((4-1)!×1!^4)	1/ 4!	952,560
32111	3	2	5	9×9!/ (9-(5-1))!	7!/ ((3-1)!×1!^3×2!^1)	1/ (3!×1!)	5,715,360
23111	3	2	5	9×9!/ (9-(5-1))!	7!/ ((2-1)!×1!^3×3!^1)	1/ (3!×1!)	3,810,240
22211	2	3	5	9×9!/ (9-(5-1))!	7!/ ((2-1)!×1!^2×2!^2)	1/ (2!×2!)	8,573,040
14111	4		5	9×9!/ (9-(5-1))!	7!/ ((1-1)!×1!^3×4!^1)	1/ (3!×1!)	952,560

13211	3	2	5	9×9!/ (9-(5-1))!	7!/ ((1-1)!×1!^2×2!^1×3!^1)	1/ (2!×1!×1!)	5,715,360
12221	2	3	5	9×9!/ (9-(5-1))!	7!/ ((1-1)!×1!^1×2!^3)	1/ (1!×3!)	2,857,680
311111	3		6	9×9!/ (9-(6-1))!	7!/ ((3-1)!×1!^5)	1/ 5!	2,857,680
221111	2	2	6	9×9!/ (9-(6-1))!	7!/ ((2-1)!×1!^4×2!^1)	1/ (4!×1!)	14,288,400
131111	3		6	9×9!/ (9-(6-1))!	7!/ ((1-1)!×1!^4×3!^1)	1/ (4!×1!)	4,762,800
122111	2	2	6	9×9!/ (9-(6-1))!	7!/ ((1-1)!×1!^3×2!^2)	1/ (3!×2!)	14,288,400
2111111	2		7	9×9!/ (9-(7-1))!	7!/ ((2-1)!×1!^6)	1/ 6!	3,810,240
1211111	2		7	9×9!/ (9-(7-1))!	7!/ ((1-1)!×1!^5×2!^1)	1/ (5!×1!)	11,430,720
11111111			8	9×9!/ (9-(8-1))!	7!/ ((1-1)!×1!^7)	1/ 7!	1,632,960

5.3 Number banding

5.3.1 Determination of bands from features

The patterns tabulated in Section 5.4.1 show how basic patterns can be decomposed so that numbers with the same attractive features as one another can be put in the same class. However, the patterns must be put into rather fewer bands, which are determined by those features. The technique for doing this, like the technique described in Section 4.3.1, can be applied in conjunction with the classification of numbers summarised in Section 4.2.4 but is more obviously fundamental to the classification summarised in Section 5.2.4 and adopted here.

There are too many combinations of lengths of longest repetitions, clusters and sequences for each to form a separate band; they need to be put together to summarise the information about them somehow. The crucial information is identified in Section 5.2.2, where a pattern is examined with reference to the following quantities:

- The quantity of distinct digits in it, which is between one and eight.
- The length of the longest repetition in it, which is between two and eight.
- The length of the longest cluster in it, which is between two and four.
- The length of the longest sequence in it, which is between five and eight (if sequences are required to have more than four ascending or descending digits, as in Section 5.2.1).

Summarising this information is feasible because the quantities are not independent of each other. In particular:

- A pattern can include a repetition of between five and eight digits ('22222457', for example) only if it contains fewer than five distinct digits.
- A pattern can include a cluster ('27452745', for example).only if it contains fewer than five distinct digits.
- A pattern can include a sequence of between five and eight digits ('12345555', for example).only if it contains more than four distinct digits.

Thus the lengths of the longest repetition, cluster and sequence in a pattern are affected by the quantity of distinct digits in the pattern, which is the length of the descriptor of the pattern. In fact the following rules summarise the information about the repetitions, clusters and sequences to increasing levels of detail:

- The 'D' rule, taking account of the length of the descriptor ("D").
- The 'DS' rule, taking account of the length of the descriptor ("D") and the length of the longest sequence ("S").
- The 'DRS' rule, taking account of the length of the descriptor ("D"), the length of the longest repetition ("R") and the length of the longest sequence ("S").
- The 'DRCS' rule, taking account of the length of the descriptor ("D"), the length of the longest repetition ("R"), the length of the longest cluster ("C") and the length of the longest sequence ("S").

The D rule can be ignored, for the reason outlined in Section 5.3.3. However, the DRCS, DRS and DS rules place usefully different levels of emphasis on pattern features that customers notice easily.

There are in fact several ways of proceeding from the D rule to the DRCS rule; for instance, the DR and DRC rules could be written by analogy with the DS and DRS rules but interchange the dependencies on repetitions, clusters and sequences. The DRCS rule is the same in all cases and is the one that takes account of the most information; the others are intended mainly to show intermediate levels of detail.

The DRCS, DRS and DS rules use information obtained by examining the basic patterns in detail according to the procedures in Section 5.2.2. They can therefore have different results for two numbers that have the same basic pattern. For instance, with the convention that a non-existent repetition or cluster is a repetition or cluster with zero digits:

- '22222447', '22224427' and '24222427' include longest repetitions with five, four and three digits respectively and longest clusters with zero, two and three digits respectively, so they could have different results from each other according to the DRCS and DRS rules (though they all have the same descriptor, '521').

- '24222459', '22522494' and '24252429' include longest repetitions with three, two and zero digits respectively and longest clusters with two, zero and three digits respectively, so they could have different results from each other according to the DRCS and DRS rules (though they all have the same descriptor, '4211').
- '27274459', '27542749' and '27452749' include longest repetitions with two, zero and zero digits respectively and longest clusters with two, two and three digits respectively, so they could have different results from each other according to the DRCS and DRS rules (though they all have the same descriptor, '22211').

The DRCS, DRS and DS can be used to put patterns together in bands such that one pattern occupies a higher band than another if it is evidently more attractive. The lengths of the longest repetition, cluster and sequence, as well as the length of the descriptor, can all be relevant to the band occupied by a pattern. For instance:

- The pattern '22272242', which has '611' as its descriptor, can be regarded highly despite including only a three-digit repetition.
- The pattern '22722727', which has '53' as its descriptor, can be regarded very highly despite including only a three-digit cluster.
- The pattern '22224927', which includes a four-digit repetition, can be regarded very highly despite having as its descriptor '5111', which is not short.
- The pattern '27452745', which includes a four-digit cluster, can be regarded very highly despite having as its descriptor '2222', which is not short.
- The pattern '24222729', which includes a three-digit repetition, can be regarded moderately highly despite having as its descriptor '5111', which is not short.
- The pattern '24252429', which includes a three-digit cluster, can be regarded moderately highly despite having as its descriptor '4211', which is not short.
- The pattern '12345657', which includes a six-digit sequence, can be regarded highly despite having as its descriptor '2111111', which is very long.
- The pattern '12345757', which includes a five-digit sequence, can be regarded moderately highly despite having as its descriptor '221111', which is long.

There are many refinements of the DRCS, DRS and DS rules that put patterns together in no more than five bands. Inevitably they lose information about the relative attractiveness of different patterns. Careful examination of them suggests that the information is best preserved by making the following assumptions:

- Repetitions with a given length and clusters with the same length affect the banding equally, if they are both taken into account (as they are according to the DRCS rule).
- Repetitions with length four and descriptors with length two affect the banding equally, if they are both taken into account (as they are according to the DRCS and DRS rules).
- Sequences with length seven and descriptors with length two affect the banding equally, if they are both taken into account (as they are according to the DRCS, DRS and DS rules).
- Longer repetitions, clusters and sequences, and shorter descriptors, lead to higher bands, except for repetitions with lengths greater than five, sequences with lengths less than five, and descriptors with lengths greater than five (provided that there are to be no more than five bands).

Table 103 defines five bands (A, B, C, D and E) formed using each of the DRCS, DRS and DS rules according to these assumptions.

Table 103 Contents of bands based on pattern features

Band	Content of band formed by…		
	DRCS	DRS	DS
A	Fitting in the band formed using the DRS rule alongside	Fitting in the band formed using the DS rule alongside or having a five-digit, six-digit or seven-digit repetition	Having one distinct digit only or having an eight-digit sequence
B	Fitting in the band formed using the DRS rule alongside or having a four-digit cluster, and not fitting in a band above	Fitting in the band formed using the DS rule alongside or having a four-digit repetition, and not fitting in a band above	Having two distinct digits only or having a seven-digit sequence, and not fitting in a band above
C	Fitting in the band formed using the DRS rule alongside or having a three-digit cluster, and not fitting in a band above	Fitting in the band formed using the DS rule alongside or having a three-digit repetition, and not fitting in a band above	Having three distinct digits only or having a six-digit sequence, and not fitting in a band above
D	Fitting in the band formed using the DRS rule alongside or having a two-digit cluster, and not fitting in a band above	Fitting in the band formed using the DS rule alongside or having a two-digit repetition, and not fitting in a band above	Having four distinct digits only or having a five-digit sequence, and not fitting in a band above
E	Not fitting in a band above	Not fitting in a band above	Not fitting in a band above

5.3.2 Quantities of numbers and classes in bands

The bands formed using each of the DRCS, DRS and DS rules constitute the DRCS, DRS and DS bands. They have more general and systematic definitions than the categories described for the benchmark countries: they are determined uniformly and consistently for a pattern by the lengths of the longest repetition, cluster and sequence in the pattern, and by the quantity of distinct digits in the pattern. They do not have obvious simplifications, given the notions of repetition, cluster and sequence. For instance, with the bands formed using the DRCS rule:

- The pattern '12345678' is in band A (instead of band E) only because its longest sequence has eight digits.
- The pattern '27227727' is in band B (instead of band D) only because it has exactly two distinct digits.
- The pattern '22284957' is in band C (instead of band E) only because its longest repetition has three digits.
- The pattern '24852479' is in band D (instead of band E) only because its longest cluster has two digits.

As the DRCS bands summarise more detail than the DRS and DS bands they should usually be preferred, though the notions of repetition, cluster and sequence might change, as discussed in Section 6.3.3.

The definitions of the DRCS, DRS and DS bands are simple enough that a number can be assigned to a band just by applying the definitions, instead of reading a table. They can be applied to the classes of numbers tabulated in Section 5.4.1 for the complete eight-digit numbering space. This is done in the table of Section 5.4.2, where there are separate columns for the DRCS, DRS and DS bands; the entries are sorted according to the DRCS, DRS and DS bands in decreasing order of priority (so the DRCS band attributed to a class is at least as high as the DRS band and the DRS band is at least as high as the DS band). Table 104 gives the capacities of the resulting bands, obtained by summing the capacities of the constituent classes.

Table 104 Capacities of bands in the complete eight-digit numbering space

Band	Capacity of band formed by….		
	DRCS	DRS	DS
A	37,006	37,006	16
B	601,708	430,348	11,578
C	7,570,586	5,403,266	698,206
D	55,167,556	47,894,742	8,615,200
E	36,623,144	46,234,638	90,675,000
Total	100,000,000	100,000,000	100,000,000

In this table, the following points about the DRCS, DRS and DS rules stand out:

- The results according to the DS rule could be combined in the higher bands, because there are very few numbers in band A.
- The differences between the results according to the DRCS and DRS rules are not very large in the higher bands; they would be reduced further if the clusters in a pattern were required to be identical, not just arbitrary permutations of each other.

Table 105 shows how many of the 178 classes of Section 5.4.2 are collected in each of the bands. The changing levels of detail summarised in the DRCS, DRS and DS bands are reflected in the migration of the classes into higher bands as the banding proceeds to the DRCS rule through the DS and DRS rules.

Table 105 Quantities of classes for bands in the complete eight-digit numbering space

Band	Quantity of classes in band formed by...		
	DRCS	DRS	DS
A	16	16	2
B	44	39	33
C	66	59	57
D	45	53	60
E	7	11	26
Total	178	178	178

The definitions of the DRCS, DRS and DS bands are applied in Section 5.4.5 to the classes of numbers tabulated in Section 5.4.4 for the restricted eight-digit numbering space. The table in Section 5.4.5 is sorted in the same manner as the table in Section 5.4.2, by following the DRCS, DRS and DS rules. Table 106 displays the capacities of the resulting bands.

Table 106 Capacities of bands in the restricted eight-digit numbering space

Band	Capacity of band formed by...		
	DRCS	DR	DS
A	33,306	33,306	15
B	537,006	387,318	10,425
C	6,813,615	4,863,135	628,474
D	49,655,844	43,105,992	7,754,886
E	32,960,229	41,610,249	81,606,200
Total	90,000,000	90,000,000	90,000,000

Table 107 shows how many of the 365 classes of Section 5.4.5 are collected in each of the bands.

Table 107 Quantities of classes for bands in the restricted eight-digit numbering space

Band	Quantity of classes in band formed by...		
	DRCS	DRS	DS
A	28	28	2
B	76	69	49
C	148	133	124
D	99	112	133
E	14	23	57
Total	365	365	365

5.3.3 Descriptors in relation to classes

The DRCS, DRS and DS bands for a pattern are determined by the lengths of the longest repetition, cluster and sequence in the pattern, and by the quantity of distinct digits in the pattern. They do not depend directly on the descriptor of the pattern; in fact two patterns that have different descriptors but the same lengths of the longest repetition, cluster and sequence, and the same quantity of distinct digits, are always in the same band as each other. For instance, with the bands formed using the DRCS, DRS or DS rules:

- The patterns '22224947' and '22224927' have different descriptors ('4211' and '5111') but are both in band B, because each includes a four-digit repetition (but no cluster or sequence).
- The patterns '49222427' and '24222729' have different descriptors ('4211' and '5111') but are both in band C, because each includes a three-digit repetition (but no cluster or sequence).
- The patterns '49224272' and '24227292' have different descriptors ('4211' and '5111') but are both in band D, because each includes a two-digit repetition (but no cluster or sequence).

Accordingly the classes of numbers specified using descriptors (along with lengths of the longest repetitions, clusters and sequences) in Section 5.4.2 can be combined into classes of numbers specified using quantities of distinct digits (along with lengths of the longest repetitions, clusters and sequences). The dependence of the classes on the related basic patterns is thereby removed in Section 5.4.3 for the complete eight-digit numbering space.

Similarly the classes of numbers specified using descriptors in Section 5.4.5 can be turned into many fewer classes specified using quantities of distinct digits in Section 5.4.6. Again the relative basic patterns become redundant.

The tables in Section 5.3.3 and Section 5.5.6 are the shortest ones that specify classes of numbers only in terms of the information that is summarised in the

DRCS, DRS and DS bands. They list 97 classes of numbers (instead of 178 for the complete numbering space and 365 for the restricted numbering space). Table 108 shows how many of these classes are collected in each of the bands. These classes are an invariant of the attractive features considered: they occur also in bands formed using the DR and DRC rules and other analogous rules.

Table 108 Quantities of classes for bands independent of related basic patterns

Band	Quantity of classes in band formed by...		
	DRCS	DRS	DS
A	13	13	2
B	27	22	21
C	31	28	26
D	22	27	31
E	4	7	17
Total	97	97	97

5.3.4 Descriptors in relation to bands

The DRCS, DRS and DS bands for a pattern depend on the lengths of the longest repetition, cluster and sequence in the pattern, as well as on the length of the descriptor of the pattern. The descriptor is the same as that of the basic pattern to which the pattern is related. Accordingly the descriptor on its own is informative, because the lengths of the longest repetition, cluster and sequence in the pattern are no more than the lengths of the longest repetition, cluster and sequence for the related basic pattern, and those lengths can be derived from the descriptor as specified in Section 5.2.5.

The determination of the DRCS, DRS and DS bands for the patterns related to a basic pattern is laborious: it entails examining the related basic pattern in detail. By contrast, the DRCS, DRS and DS bands for the related basic pattern can be derived immediately from the descriptor. They can provide some guidance about the DRCS, DRS and DS bands for the patterns related to the basic pattern.

Unfortunately this guidance is too simplistic on its own: longer repetitions and clusters tend to occur in patterns with shorter descriptors (such as '62' and '521') but longer sequences tend to occur in patterns with longer descriptors (such as '22211' and '221111'), so both basic patterns with shorter descriptors and basic patterns with longer descriptors occupy higher bands.

For this reason the D rule of Section 5.3.1, which takes into account the length of the descriptor but not the length of the longest sequence, is also too simplistic. The information provided by it has to be augmented in increasing levels of detail by proceeding from the D rule to the DRCS rule through the DS and DRS rules or the DR and DRC rules (for example); whichever route is taken, the DRCS rule remains the preferred end point, as it allows the most detail to be summarised.

5.4 Analysis of the eight-digit numbering space

5.4.1 Capacities for patterns representing each number in the complete eight-digit numbering space once only and separating patterns with attractive features and different related basic patterns to avoid overlaps

Table 109 Capacities of eight-digit patterns containing occurrences of exactly one letter

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
8	8			10

Table 110 Capacities of eight-digit patterns containing occurrences of exactly two letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
44	4	4		90
44	4	3		90
44	4	2		90
44	4			90
44		4		90
44	2	3		180
44	2	2		180
44	3	3		270
44	3	4		360
44	3	2		630
44	2	4		1,080
53	5	3		180
53	5			180
53	3	2		180
53	4	3		540
53	4	2		540
53	2	3		1,440
53	3	3		1,980
62	6	4		90
62	2	4		90
62	6			180
62	5	4		180

Enumerating patterns

62	4	4		270
62	5	2		360
62	4	3		540
62	3	4		810
71	7			180
71	6			180
71	5			180
71	4			180

Table 111 Capacities of eight-digit patterns containing occurrences of exactly three letters

Pattern				Capacity
Related basic pattern	**Length of the longest repetition**	**Length of the longest cluster**	**Length of the longest sequence**	
332		2		1,440
332	3	3		5,760
332	2			5,760
332	3			10,080
332	3	2		23,040
332		3		25,200
332	2	2		63,360
332	2	3		66,960
422	4	4		1,440
422	4	3		1,440
422		2		2,160
422	4	2		2,880
422	2			3,600
422	4			5,040
422	3	4		5,760
422		4		6,480
422	3			8,640
422	3	3		11,520
422	3	2		17,280
422	2	3		17,280
422	2	2		29,520
422	2	4		38,160
431	2			1,440
431	4	3		4,320
431		3		4,320
431		2		5,760
431	4			10,080
431	3	3		10,080
431	3			14,400

431	3	2		41,760
431	2	3		46,080
431	2	2		63,360
521	5	3		1,440
521	4	3		4,320
521	5			7,200
521	3			8,640
521	4			10,080
521	4	2		11,520
521	2	2		12,960
521	3	2		18,720
521	2	3		21,600
521	3	3		24,480
611	2			720
611	6			2,160
611	5			4,320
611	4			6,480
611	3			6,480

Table 112 Capacities of eight-digit patterns containing occurrences of exactly four letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
2222	2	3		15,120
2222	2	4		30,240
2222		2		38,640
2222				52,080
2222		4		90,720
2222	2	2		146,160
2222	2			156,240
3221	3	3		20,160
3221				90,720
3221	3	2		151,200
3221	3			282,240
3221		2		413,280
3221	2	3		423,360
3221		3		463,680
3221	2			897,120
3221	2	2		1,491,840
3311	3	2		60,480
3311	3			211,680
3311		2		231,840

Enumerating patterns

3311	2			241,920
3311	2	2		665,280
4211	4	3		15,120
4211		3		15,120
4211	3	3		30,240
4211		2		120,960
4211	4			136,080
4211	2	3		211,680
4211	3	2		211,680
4211	3			362,880
4211	2			378,000
4211	2	2		635,040
5111	5			20,160
5111	4			60,480
5111	2			80,640
5111	3			120,960

Table 113 Capacities of eight-digit patterns containing occurrences of exactly five letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
22211	2	3	5	48
22211	2	2	5	336
22211	2		5	408
22211		3	5	432
22211		2	5	816
22211			5	840
22211	2	3		181,392
22211		3		906,768
22211		2		1,904,304
22211	2	2		2,086,224
22211				2,781,240
22211	2			4,837,992
32111	3	2	5	24
32111	3		5	168
32111			5	336
32111	2	2	5	600
32111		2	5	648
32111	2		5	1,104
32111	3	2		120,936
32111	3			1,693,272
32111				1,814,064

32111	2	2		2,902,440
32111		2		3,023,352
32111	2			7,377,456
41111	4		5	24
41111	2		5	96
41111	3		5	120
41111	4			151,176
41111				151,200
41111	3			604,680
41111	2			1,209,504

Table 114 Capacities of eight-digit patterns containing occurrences of exactly six letters

Pattern Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	Capacity
221111	2	2	6	20
221111	2		6	170
221111		2	6	190
221111			6	520
221111	2	2	5	320
221111		2	5	2,740
221111	2		5	3,980
221111			5	9,160
221111	2	2		755,660
221111		2		3,777,070
221111	2			12,847,850
221111				14,354,320
311111	3		6	20
311111			6	40
311111	2		6	120
311111	3		5	320
311111			5	1,240
311111	2		5	1,920
311111	3			906,860
311111				3,022,720
311111	2			4,533,960

Table 115 Capacities of eight-digit patterns containing occurrences of exactly seven letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
2111111	2		7	16
2111111			7	96
2111111	2		6	208
2111111			6	1,128
2111111	2		5	2,416
2111111			5	11,856
2111111	2			4,230,960
2111111				12,687,720

Table 116 Capacities of eight-digit patterns containing occurrences of exactly eight letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
11111111			8	6
11111111			7	36
11111111			6	270
11111111			5	2,208
11111111				1,811,880

5.4.2 Bands for patterns representing each number in the complete eight-digit numbering space once only and separating patterns with attractive features and different related basic patterns to avoid overlaps

Table 117 Bands for eight-digit patterns with repetitions, clusters and sequences separated out

Band assessed by...			Pattern				Capacity
DRCS	DRS	DS	Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
A	A	A	8	8			10
A	A	A	11111111			8	6
A	A	B	53	5	3		180

A	A	B	53	5			180
A	A	B	62	6	4		90
A	A	B	62	6			180
A	A	B	62	5	4		180
A	A	B	62	5	2		360
A	A	B	71	7			180
A	A	B	71	6			180
A	A	B	71	5			180
A	A	C	521	5	3		1,440
A	A	C	521	5			7,200
A	A	C	611	6			2,160
A	A	C	611	5			4,320
A	A	D	5111	5			20,160
B	B	B	44	4	4		90
B	B	B	44	4	3		90
B	B	B	44	4	2		90
B	B	B	44	4			90
B	B	B	44	3	4		360
B	B	B	44	3	3		270
B	B	B	44	3	2		630
B	B	B	44	2	4		1,080
B	B	B	44	2	3		180
B	B	B	44	2	2		180
B	B	B	44		4		90
B	B	B	53	4	3		540
B	B	B	53	4	2		540
B	B	B	53	3	3		1,980
B	B	B	53	3	2		180
B	B	B	53	2	3		1,440
B	B	B	62	4	4		270
B	B	B	62	4	3		540
B	B	B	62	3	4		810
B	B	B	62	2	4		90
B	B	B	71	4			180
B	B	B	2111111	2		7	16
B	B	B	2111111			7	96
B	B	B	11111111			7	36
B	B	C	422	4	4		1,440
B	B	C	422	4	3		1,440
B	B	C	422	4	2		2,880
B	B	C	422	4			5,040
B	B	C	431	4	3		4,320
B	B	C	431	4			10,080
B	B	C	521	4	3		4,320
B	B	C	521	4	2		11,520
B	B	C	521	4			10,080

B	B	C	611	4			6,480
B	B	D	4211	4	3		15,120
B	B	D	4211	4			136,080
B	B	D	5111	4			60,480
B	B	D	41111	4		5	24
B	B	E	41111	4			151,176
B	C	C	422	3	4		5,760
B	C	C	422	2	4		38,160
B	C	C	422		4		6,480
B	D	D	2222	2	4		30,240
B	D	D	2222		4		90,720
C	C	C	332	3	3		5,760
C	C	C	332	3	2		23,040
C	C	C	332	3			10,080
C	C	C	332	2	3		66,960
C	C	C	332	2	2		63,360
C	C	C	332	2			5,760
C	C	C	332		3		25,200
C	C	C	332		2		1,440
C	C	C	422	3	3		11,520
C	C	C	422	3	2		17,280
C	C	C	422	3			8,640
C	C	C	422	2	3		17,280
C	C	C	422	2	2		29,520
C	C	C	422	2			3,600
C	C	C	422		2		2,160
C	C	C	431	3	3		10,080
C	C	C	431	3	2		41,760
C	C	C	431	3			14,400
C	C	C	431	2	3		46,080
C	C	C	431	2	2		63,360
C	C	C	431	2			1,440
C	C	C	431		3		4,320
C	C	C	431		2		5,760
C	C	C	521	3	3		24,480
C	C	C	521	3	2		18,720
C	C	C	521	3			8,640
C	C	C	521	2	3		21,600
C	C	C	521	2	2		12,960
C	C	C	611	3			6,480
C	C	C	611	2			720
C	C	C	221111	2	2	6	20
C	C	C	221111	2		6	170
C	C	C	221111		2	6	190
C	C	C	221111			6	520
C	C	C	311111	3		6	20

C	C	C	311111	2		6	120
C	C	C	311111			6	40
C	C	C	2111111	2		6	208
C	C	C	2111111			6	1,128
C	C	C	11111111			6	270
C	C	D	3221	3	3		20,160
C	C	D	3221	3	2		151,200
C	C	D	3221	3			282,240
C	C	D	3311	3	2		60,480
C	C	D	3311	3			211,680
C	C	D	4211	3	3		30,240
C	C	D	4211	3	2		211,680
C	C	D	4211	3			362,880
C	C	D	5111	3			120,960
C	C	D	32111	3	2	5	24
C	C	D	32111	3		5	168
C	C	D	311111	3		5	320
C	C	E	32111	3	2		120,936
C	C	E	32111	3			1,693,272
C	C	E	41111	3			604,680
C	C	E	311111	3			906,860
C	D	D	2222	2	3		15,120
C	D	D	3221	2	3		423,360
C	D	D	3221		3		463,680
C	D	D	4211	2	3		211,680
C	D	D	4211		3		15,120
C	D	D	22211	2	3	5	48
C	D	D	22211		3	5	432
C	D	D	41111	3		5	120
C	D	E	22211	2	3		181,392
C	E	E	22211		3		906,768
D	D	D	2222	2	2		146,160
D	D	D	2222	2			156,240
D	D	D	2222		2		38,640
D	D	D	2222				52,080
D	D	D	3221	2	2		1,491,840
D	D	D	3221	2			897,120
D	D	D	3221		2		413,280
D	D	D	3221				90,720
D	D	D	3311	2	2		665,280
D	D	D	3311	2			241,920
D	D	D	3311		2		231,840
D	D	D	4211	2	2		635,040
D	D	D	4211	2			378,000
D	D	D	4211		2		120,960
D	D	D	5111	2			80,640

D	D	D	22211	2	2	5	336
D	D	D	22211	2		5	408
D	D	D	22211		2	5	816
D	D	D	22211			5	840
D	D	D	32111	2	2	5	600
D	D	D	32111	2		5	1,104
D	D	D	32111		2	5	648
D	D	D	32111			5	336
D	D	D	41111	2		5	96
D	D	D	221111	2	2	5	320
D	D	D	221111	2		5	3,980
D	D	D	221111		2	5	2,740
D	D	D	221111			5	9,160
D	D	D	311111	2		5	1,920
D	D	D	311111			5	1,240
D	D	D	2111111	2		5	2,416
D	D	D	2111111			5	11,856
D	D	D	11111111			5	2,208
D	D	E	22211	2	2		2,086,224
D	D	E	22211	2			4,837,992
D	D	E	32111	2	2		2,902,440
D	D	E	32111	2			7,377,456
D	D	E	41111	2			1,209,504
D	D	E	221111	2	2		755,660
D	D	E	221111	2			12,847,850
D	D	E	311111	2			4,533,960
D	D	E	2111111	2			4,230,960
D	E	E	22211		2		1,904,304
D	E	E	32111		2		3,023,352
D	E	E	221111		2		3,777,070
E	E	E	22211				2,781,240
E	E	E	32111				1,814,064
E	E	E	41111				151,200
E	E	E	221111				14,354,320
E	E	E	311111				3,022,720
E	E	E	2111111				12,687,720
E	E	E	11111111				1,811,880

5.4.3 Bands for patterns representing each number in the complete eight-digit numbering space once only and separating patterns with attractive features and different quantities of distinct digits to avoid overlaps

Table 118 Bands for eight-digit patterns with repetitions, clusters and sequences separated out

Band assessed by...			Pattern				Capacity
DRCS	DRS	DS	Quantity of distinct digits	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
A	A	A	1	8			10
A	A	A	8			8	6
A	A	B	2	7			180
A	A	B	2	6	4		90
A	A	B	2	6			360
A	A	B	2	5	4		180
A	A	B	2	5	3		180
A	A	B	2	5	2		360
A	A	B	2	5			360
A	A	C	3	6			2,160
A	A	C	3	5	3		1,440
A	A	C	3	5			11,520
A	A	D	4	5			20,160
B	B	B	2	4	4		360
B	B	B	2	4	3		1,170
B	B	B	2	4	2		630
B	B	B	2	4			270
B	B	B	2	3	4		1,170
B	B	B	2	3	3		2,250
B	B	B	2	3	2		810
B	B	B	2	2	4		1,170
B	B	B	2	2	3		1,620
B	B	B	2	2	2		180
B	B	B	2		4		90
B	B	B	7	2		7	16
B	B	B	7			7	96
B	B	B	8			7	36
B	B	C	3	4	4		1,440
B	B	C	3	4	3		10,080
B	B	C	3	4	2		14,400
B	B	C	3	4			31,680
B	B	D	4	4	3		15,120
B	B	D	4	4			196,560
B	B	D	5	4		5	24
B	B	E	5	4			151,176

Enumerating patterns

B	C	C	3	3	4		5,760
B	C	C	3	2	4		38,160
B	C	C	3		4		6,480
B	D	D	4	2	4		30,240
B	D	D	4		4		90,720
C	C	C	3	3	3		51,840
C	C	C	3	3	2		100,800
C	C	C	3	3			48,240
C	C	C	3	2	3		151,920
C	C	C	3	2	2		169,200
C	C	C	3	2			11,520
C	C	C	3		3		29,520
C	C	C	3		2		9,360
C	C	C	6	3		6	20
C	C	C	6	2	2	6	20
C	C	C	6	2		6	290
C	C	C	6		2	6	190
C	C	C	6			6	560
C	C	C	7	2		6	208
C	C	C	7			6	1,128
C	C	C	8			6	270
C	C	D	4	3	3		50,400
C	C	D	4	3	2		423,360
C	C	D	4	3			977,760
C	C	D	5	3	2	5	24
C	C	D	5	3		5	288
C	C	D	6	3		5	320
C	C	E	5	3	2		120,936
C	C	E	5	3			2,297,952
C	C	E	6	3			906,860
C	D	D	4	2	3		650,160
C	D	D	4		3		478,800
C	D	D	5	2	3	5	48
C	D	D	5		3	5	432
C	D	E	5	2	3		181,392
C	E	E	5		3		906,768
D	D	D	4	2	2		2,938,320
D	D	D	4	2			1,753,920
D	D	D	4		2		804,720
D	D	D	4				142,800
D	D	D	5	2	2	5	936
D	D	D	5	2		5	1,608
D	D	D	5		2	5	1,464
D	D	D	5			5	1,176
D	D	D	6	2	2	5	320
D	D	D	6	2		5	5,900

D	D	D	6		2	5	2,740
D	D	D	6			5	10,400
D	D	D	7	2		5	2,416
D	D	D	7			5	11,856
D	D	D	8			5	2,208
D	D	E	5	2	2		4,988,664
D	D	E	5	2			13,424,952
D	D	E	6	2	2		755,660
D	D	E	6	2			17,381,810
D	D	E	7	2			4,230,960
D	E	E	5		2		4,927,656
D	E	E	6		2		3,777,070
E	E	E	5				4,746,504
E	E	E	6				17,377,040
E	E	E	7				12,687,720
E	E	E	8				1,811,880

5.4.4 Capacities for patterns representing each number in the restricted eight-digit numbering space once only and separating patterns with attractive features and different related basic patterns to avoid overlaps

Table 119 Capacities of eight-digit patterns containing occurrences of exactly one letter

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
8	8			9

Table 120 Capacities of eight-digit patterns containing occurrences of exactly two letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
17	6			81
26	6			81
26	5	4		81
26	5	2		81
26	4	4		81
26	4	2		81
26	3	4		162
35	5	3		81

35	5			81
35	3	2		81
35	4	2		162
35	4	3		243
35	2	3		486
35	3	3		567
44	4	4		81
44	4	3		81
44	4	2		81
44	4			81
44	2	3		81
44		4		81
44	3	3		162
44	2	2		243
44	3	4		324
44	3	2		648
44	2	4		972
53	5	3		81
53	5			81
53	3	2		81
53	4	3		243
53	4	2		324
53	2	3		810
53	3	3		1,215
62	6	4		81
62	6			81
62	5	4		81
62	2	4		81
62	4	4		162
62	5	2		243
62	4	3		405
62	3	4		567
71	6			81
71	7			162
71	5			162
71	4			162

Table 121 Capacities of eight-digit patterns containing occurrences of exactly three letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
143	4	3		648
143	4			1,296
143	3	3		1,296
143	3			1,296
143		2		1,296
143	2	3		3,888
143	3	2		5,184
143	2	2		7,776
152	4	3		648
152	5			1,296
152	3	3		1,296
152	3			1,296
152	4			2,592
152	3	2		2,592
152	2	3		3,888
161	6			648
161	5			1,296
161	4			1,296
161	3			1,296
233	2			1,296
233	3			1,944
233	3	3		2,592
233	3	2		5,184
233		3		5,832
233	2	2		13,608
233	2	3		14,904
242	4	3		648
242	4	4		1,296
242	4	2		1,296
242	2			1,296
242		2		1,296
242	4			2,592
242	3	4		3,888
242	3	3		4,536
242	3			4,536
242		3		4,536
242	3	2		6,480
242	2	3		7,776
242	2	4		13,608

242	2	2		14,256
251	5	3		648
251	4	3		1,296
251	3			1,296
251	5			1,944
251	4	2		2,592
251	4			2,592
251	2	2		2,592
251	3	3		4,536
251	3	2		4,536
251	2	3		5,184
332		2		1,296
332	3	3		2,592
332	2			3,888
332	3			7,128
332	3	2		15,552
332		3		16,848
332	2	2		43,416
332	2	3		45,360
341	2			648
341	4	3		1,944
341	3			2,592
341	2			2,592
341	4			3,888
341	3	3		3,888
341	3			5,184
341	3	2		12,312
341	2	3		12,960
341	2	2		22,032
422	4	3		648
422		2		648
422	4	2		1,296
422	3	4		1,296
422		4		1,296
422	4			1,944
422	2			1,944
422	3			3,240
422	3	3		5,832
422	2	3		7,776
422	3	2		9,072
422	2	2		12,312
422	2	4		20,736
431	2			648
431	4	3		1,296
431		3		1,296
431		2		1,296

431	4			3,888
431	3	3		3,888
431	3			6,480
431	3	2		20,088
431	2	3		24,624
431	2	2		27,216
521	5	3		648
521	4	3		1,944
521	5			3,240
521	4			3,888
521	3			5,184
521	4	2		7,776
521	2	2		9,072
521	3	2		9,720
521	2	3		10,368
521	3	3		16,200
611	2			648
611	6			1,296
611	5			2,592
611	4			4,536
611	3			4,536

Table 122 Capacities of eight-digit patterns containing occurrences of exactly four letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
1322	3	2		9,072
1322				13,608
1322	2	3		27,216
1322	3			45,360
1322		2		45,360
1322		3		54,432
1322	2			117,936
1322	2	2		163,296
1331	3	2		18,144
1331	3			49,896
1331		2		49,896
1331	2			63,504
1331	2	2		136,080
1421	4	3		4,536
1421	3	3		9,072
1421	3	2		27,216

1421	2	3		27,216
1421	4			36,288
1421		2		45,360
1421	3			99,792
1421	2	2		108,864
1421	2			117,936
1511	5			9,072
1511	4			22,680
1511	2			27,216
1511	3			36,288
2222	2	3		9,072
2222	2	4		27,216
2222		2		35,532
2222				46,116
2222		4		81,648
2222	2	2		135,324
2222	2			141,372
2321	3	3		18,144
2321				58,968
2321	3	2		77,112
2321	3			122,472
2321		2		181,440
2321	2	3		190,512
2321		3		226,800
2321	2			371,952
2321	2	2		657,720
2411	4	3		4,536
2411	3	3		9,072
2411		3		9,072
2411	4			36,288
2411		2		40,824
2411	2	3		45,360
2411	3	2		49,896
2411	2			63,504
2411	3			77,112
2411	2	2		140,616
3221				9,072
3221	3	2		49,896
3221	3			86,184
3221		3		136,080
3221		2		145,152
3221	2	3		163,296
3221	2			317,520
3221	2	2		521,640
3311	3	2		36,288
3311	3			140,616

3311	2			154,224
3311		2		158,760
3311	2	2		462,672
4211	4	3		4,536
4211		3		4,536
4211	3	3		9,072
4211		2		22,680
4211	4			49,896
4211	3	2		113,400
4211	2	3		117,936
4211	3			149,688
4211	2			158,760
4211	2	2		322,056
5111	5			9,072
5111	4			31,752
5111	2			45,360
5111	3			72,576

Table 123 Capacities of eight-digit patterns containing occurrences of exactly five letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
12221	2	3	5	24
12221	2	2	5	34
12221		3	5	48
12221	2		5	80
12221			5	172
12221		2	5	194
12221	2	3		54,408
12221		3		108,816
12221	2	2		272,126
12221		2		489,694
12221				680,228
12221	2			1,251,856
13211	3	2	5	12
13211	3		5	57
13211			5	136
13211	2	2	5	142
13211		2	5	170
13211	2		5	311
13211	3	2		54,420
13211	3			598,695

13211		2		761,878
13211	2	2		816,338
13211				979,640
13211	2			2,503,561
14111	4		5	12
14111	2		5	33
14111	3		5	47
14111	4			81,636
14111				108,864
14111	3			272,113
14111	2			489,855
22211	2	3	5	24
22211	2	2	5	290
22211	2		5	304
22211		3	5	377
22211		2	5	595
22211			5	630
22211	2	3		108,840
22211		3		707,239
22211		2		1,224,125
22211	2	2		1,605,454
22211				1,822,842
22211	2			3,102,320
23111	3		5	23
23111	2	2	5	94
23111			5	130
23111	2		5	173
23111		2	5	320
23111	3	2		27,216
23111				326,462
23111	3			408,217
23111	2	2		625,874
23111		2		897,808
23111	2			1,523,923
32111	3	2	5	11
32111			5	60
32111	3		5	77
32111		2	5	132
32111	2	2	5	349
32111	2		5	575
32111	3	2		27,205
32111				326,532
32111	3			517,027
32111		2		1,061,292
32111	2	2		1,169,939
32111	2			2,612,161

41111	4		5	11
41111	2		5	58
41111	3		5	70
41111				27,216
41111	4			54,421
41111	3			272,090
41111	2			598,694

Table 124 Capacities of eight-digit patterns containing occurrences of exactly six letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
122111	2	2	6	10
122111		2	6	50
122111	2		6	66
122111	2	2	5	155
122111			6	254
122111		2	5	655
122111	2		5	1,538
122111			5	4,412
122111	2	2		408,075
122111		2		1,224,015
122111	2			5,577,676
122111				7,071,494
131111	3		6	10
131111			6	36
131111	2		6	49
131111	3		5	155
131111	2		5	757
131111			5	893
131111	3			544,155
131111				1,904,191
131111	2			2,312,554
221111	2	2	6	9
221111	2		6	91
221111		2	6	138
221111	2	2	5	147
221111			6	242
221111		2	5	1,989
221111	2		5	2,103
221111			5	4,126
221111	2	2		272,004

221111		2		2,175,153
221111				5,847,072
221111	2			5,985,326
311111	3		6	9
311111	2		6	68
311111	3		5	142
311111			5	240
311111	2		5	1,069
311111	3			272,009
311111				816,240
311111	2			1,767,903

Table 125 Capacities of eight-digit patterns containing occurrences of exactly seven letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
1211111	2		7	8
1211111			7	82
1211111	2		6	100
1211111			6	748
1211111	2		5	1,336
1211111			5	7,998
1211111	2			2,720,156
1211111				8,700,292
2111111	2		7	7
2111111			7	8
2111111	2		6	90
2111111			6	296
2111111	2		5	839
2111111			5	2,844
2111111	2			1,087,704
2111111				2,718,452

Table 126 Capacities of eight-digit patterns containing occurrences of exactly eight letters

Pattern				Capacity
Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
11111111			8	6
11111111			7	33
11111111			6	240
11111111			5	1,977
11111111				1,630,704

5.4.5 Bands for patterns representing each number in the restricted eight-digit numbering space once only and separating patterns with attractive features and different related basic patterns to avoid overlaps

Table 127 Bands for eight-digit patterns with repetitions, clusters and sequences separated out

Band assessed by...			Pattern				Capacity
DRCS	DRS	DS	Related basic pattern	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
A	A	A	8	8			9
A	A	A	11111111			8	6
A	A	B	17	6			81
A	A	B	26	6			81
A	A	B	26	5	4		81
A	A	B	26	5	2		81
A	A	B	35	5	3		81
A	A	B	35	5			81
A	A	B	53	5	3		81
A	A	B	53	5			81
A	A	B	62	6	4		81
A	A	B	62	6			81
A	A	B	62	5	4		81
A	A	B	62	5	2		243
A	A	B	71	7			162
A	A	B	71	6			81
A	A	B	71	5			162
A	A	C	152	5			1,296
A	A	C	161	6			648
A	A	C	161	5			1,296
A	A	C	251	5	3		648
A	A	C	251	5			1,944

A	A	C	521	5	3		648
A	A	C	521	5			3,240
A	A	C	611	6			1,296
A	A	C	611	5			2,592
A	A	D	1511	5			9,072
A	A	D	5111	5			9,072
B	B	B	26	4	4		81
B	B	B	26	4	2		81
B	B	B	26	3	4		162
B	B	B	35	4	3		243
B	B	B	35	4	2		162
B	B	B	35	3	3		567
B	B	B	35	3	2		81
B	B	B	35	2	3		486
B	B	B	44	4	4		81
B	B	B	44	4	3		81
B	B	B	44	4	2		81
B	B	B	44	4			81
B	B	B	44	3	4		324
B	B	B	44	3	3		162
B	B	B	44	3	2		648
B	B	B	44	2	4		972
B	B	B	44	2	3		81
B	B	B	44	2	2		243
B	B	B	44		4		81
B	B	B	53	4	3		243
B	B	B	53	4	2		324
B	B	B	53	3	3		1,215
B	B	B	53	3	2		81
B	B	B	53	2	3		810
B	B	B	62	4	4		162
B	B	B	62	4	3		405
B	B	B	62	3	4		567
B	B	B	62	2	4		81
B	B	B	71	4			162
B	B	B	1211111	2		7	8
B	B	B	1211111			7	82
B	B	B	2111111	2		7	7
B	B	B	2111111			7	8
B	B	B	11111111			7	33
B	B	C	143	4	3		648
B	B	C	143	4			1,296
B	B	C	152	4	3		648
B	B	C	152	4			2,592
B	B	C	161	4			1,296
B	B	C	242	4	4		1,296

B	B	C	242	4	3		648
B	B	C	242	4	2		1,296
B	B	C	242	4			2,592
B	B	C	251	4	3		1,296
B	B	C	251	4	2		2,592
B	B	C	251	4			2,592
B	B	C	341	4	3		1,944
B	B	C	341	4			3,888
B	B	C	422	4	3		648
B	B	C	422	4	2		1,296
B	B	C	422	4			1,944
B	B	C	431	4	3		1,296
B	B	C	431	4			3,888
B	B	C	521	4	3		1,944
B	B	C	521	4	2		7,776
B	B	C	521	4			3,888
B	B	C	611	4			4,536
B	B	D	1421	4	3		4,536
B	B	D	1421	4			36,288
B	B	D	1511	4			22,680
B	B	D	2411	4	3		4,536
B	B	D	2411	4			36,288
B	B	D	4211	4	3		4,536
B	B	D	4211	4			49,896
B	B	D	5111	4			31,752
B	B	D	14111	4		5	12
B	B	D	41111	4		5	11
B	B	E	14111	4			81,636
B	B	E	41111	4			54,421
B	C	C	242	3	4		3,888
B	C	C	242	2	4		13,608
B	C	C	422	3	4		1,296
B	C	C	422	2	4		20,736
B	C	C	422		4		1,296
B	D	D	2222	2	4		27,216
B	D	D	2222		4		81,648
C	C	C	143	3	3		1,296
C	C	C	143	3	2		5,184
C	C	C	143	3			1,296
C	C	C	143	2	3		3,888
C	C	C	143	2	2		7,776
C	C	C	143		2		1,296
C	C	C	152	3	3		1,296
C	C	C	152	3	2		2,592
C	C	C	152	3			1,296
C	C	C	152	2	3		3,888

C	C	C	161	3			1,296
C	C	C	233	3	3		2,592
C	C	C	233	3	2		5,184
C	C	C	233	3			1,944
C	C	C	233	2	3		14,904
C	C	C	233	2	2		13,608
C	C	C	233	2			1,296
C	C	C	233		3		5,832
C	C	C	242	3	3		4,536
C	C	C	242	3	2		6,480
C	C	C	242	3			4,536
C	C	C	242	2	3		7,776
C	C	C	242	2	2		14,256
C	C	C	242	2			1,296
C	C	C	242		3		4,536
C	C	C	242		2		1,296
C	C	C	251	3	3		4,536
C	C	C	251	3	2		4,536
C	C	C	251	3			1,296
C	C	C	251	2	3		5,184
C	C	C	251	2	2		2,592
C	C	C	332	3	3		2,592
C	C	C	332	3	2		15,552
C	C	C	332	3			7,128
C	C	C	332	2	3		45,360
C	C	C	332	2	2		43,416
C	C	C	332	2			3,888
C	C	C	332		3		16,848
C	C	C	332		2		1,296
C	C	C	341	3	3		3,888
C	C	C	341	3	2		12,312
C	C	C	341	3			5,184
C	C	C	341	2	3		12,960
C	C	C	341	2	2		22,032
C	C	C	341	2			648
C	C	C	341		3		2,592
C	C	C	341		2		2,592
C	C	C	422	3	3		5,832
C	C	C	422	3	2		9,072
C	C	C	422	3			3,240
C	C	C	422	2	3		7,776
C	C	C	422	2	2		12,312
C	C	C	422	2			1,944
C	C	C	422		2		648
C	C	C	431	3	3		3,888
C	C	C	431	3	2		20,088

C	C	C	431	3			6,480
C	C	C	431	2	3		24,624
C	C	C	431	2	2		27,216
C	C	C	431	2			648
C	C	C	431		3		1,296
C	C	C	431		2		1,296
C	C	C	521	3	3		16,200
C	C	C	521	3	2		9,720
C	C	C	521	3			5,184
C	C	C	521	2	3		10,368
C	C	C	521	2	2		9,072
C	C	C	611	3			4,536
C	C	C	611	2			648
C	C	C	122111	2	2	6	10
C	C	C	122111	2		6	66
C	C	C	122111		2	6	50
C	C	C	122111			6	254
C	C	C	131111	3		6	10
C	C	C	131111	2		6	49
C	C	C	131111			6	36
C	C	C	221111	2	2	6	9
C	C	C	221111	2		6	91
C	C	C	221111		2	6	138
C	C	C	221111			6	242
C	C	C	311111	3		6	9
C	C	C	311111	2		6	68
C	C	C	1211111	2		6	100
C	C	C	1211111			6	748
C	C	C	2111111	2		6	90
C	C	C	2111111			6	296
C	C	C	11111111			6	240
C	C	D	1322	3	2		9,072
C	C	D	1322	3			45,360
C	C	D	1331	3	2		18,144
C	C	D	1331	3			49,896
C	C	D	1421	3	3		9,072
C	C	D	1421	3	2		27,216
C	C	D	1421	3			99,792
C	C	D	1511	3			36,288
C	C	D	2321	3	3		18,144
C	C	D	2321	3	2		77,112
C	C	D	2321	3			122,472
C	C	D	2411	3	3		9,072
C	C	D	2411	3	2		49,896
C	C	D	2411	3			77,112
C	C	D	3221	3	2		49,896

C	C	D	3221	3			86,184
C	C	D	3311	3	2		36,288
C	C	D	3311	3			140,616
C	C	D	4211	3	3		9,072
C	C	D	4211	3	2		113,400
C	C	D	4211	3			149,688
C	C	D	5111	3			72,576
C	C	D	13211	3	2	5	12
C	C	D	13211	3		5	57
C	C	D	14111	3		5	47
C	C	D	23111	3		5	23
C	C	D	32111	3	2	5	11
C	C	D	32111	3		5	77
C	C	D	41111	3		5	70
C	C	D	131111	3		5	155
C	C	D	311111	3		5	142
C	C	E	13211	3	2		54,420
C	C	E	13211	3			598,695
C	C	E	14111	3			272,113
C	C	E	23111	3	2		27,216
C	C	E	23111	3			408,217
C	C	E	32111	3	2		27,205
C	C	E	32111	3			517,027
C	C	E	41111	3			272,090
C	C	E	131111	3			544,155
C	C	E	311111	3			272,009
C	D	D	1322	2	3		27,216
C	D	D	1322		3		54,432
C	D	D	1421	2	3		27,216
C	D	D	2222	2	3		9,072
C	D	D	2321	2	3		190,512
C	D	D	2321		3		226,800
C	D	D	2411	2	3		45,360
C	D	D	2411		3		9,072
C	D	D	3221	2	3		163,296
C	D	D	3221		3		136,080
C	D	D	4211	2	3		117,936
C	D	D	4211		3		4,536
C	D	D	12221	2	3	5	24
C	D	D	12221		3	5	48
C	D	D	22211	2	3	5	24
C	D	D	22211		3	5	377
C	D	E	12221	2	3		54,408
C	D	E	22211	2	3		108,840
C	E	E	12221		3		108,816
C	E	E	22211		3		707,239

D	D	D	1322	2	2		163,296
D	D	D	1322	2			117,936
D	D	D	1322		2		45,360
D	D	D	1322				13,608
D	D	D	1331	2	2		136,080
D	D	D	1331	2			63,504
D	D	D	1331		2		49,896
D	D	D	1421	2	2		108,864
D	D	D	1421	2			117,936
D	D	D	1421		2		45,360
D	D	D	1511	2			27,216
D	D	D	2222	2	2		135,324
D	D	D	2222	2			141,372
D	D	D	2222		2		35,532
D	D	D	2222				46,116
D	D	D	2321	2	2		657,720
D	D	D	2321	2			371,952
D	D	D	2321		2		181,440
D	D	D	2321				58,968
D	D	D	2411	2	2		140,616
D	D	D	2411	2			63,504
D	D	D	2411		2		40,824
D	D	D	3221	2	2		521,640
D	D	D	3221	2			317,520
D	D	D	3221		2		145,152
D	D	D	3221				9,072
D	D	D	3311	2	2		462,672
D	D	D	3311	2			154,224
D	D	D	3311		2		158,760
D	D	D	4211	2	2		322,056
D	D	D	4211	2			158,760
D	D	D	4211		2		22,680
D	D	D	5111	2			45,360
D	D	D	12221	2	2	5	34
D	D	D	12221	2		5	80
D	D	D	12221		2	5	194
D	D	D	12221			5	172
D	D	D	13211	2	2	5	142
D	D	D	13211	2		5	311
D	D	D	13211		2	5	170
D	D	D	13211			5	136
D	D	D	14111	2		5	33
D	D	D	22211	2	2	5	290
D	D	D	22211	2		5	304
D	D	D	22211		2	5	595
D	D	D	22211			5	630

D	D	D	23111	2	2	5	94
D	D	D	23111	2		5	173
D	D	D	23111		2	5	320
D	D	D	23111			5	130
D	D	D	32111	2	2	5	349
D	D	D	32111	2		5	575
D	D	D	32111		2	5	132
D	D	D	32111			5	60
D	D	D	41111	2		5	58
D	D	D	122111	2	2	5	155
D	D	D	122111	2		5	1,538
D	D	D	122111		2	5	655
D	D	D	122111			5	4,412
D	D	D	131111	2		5	757
D	D	D	131111			5	893
D	D	D	221111	2	2	5	147
D	D	D	221111	2		5	2,103
D	D	D	221111		2	5	1,989
D	D	D	221111			5	4,126
D	D	D	311111	2		5	1,069
D	D	D	311111			5	240
D	D	D	1211111	2		5	1,336
D	D	D	1211111			5	7,998
D	D	D	2111111	2		5	839
D	D	D	2111111			5	2,844
D	D	D	11111111			5	1,977
D	D	E	12221	2	2		272,126
D	D	E	12221	2			1,251,856
D	D	E	13211	2	2		816,338
D	D	E	13211	2			2,503,561
D	D	E	14111	2			489,855
D	D	E	22211	2	2		1,605,454
D	D	E	22211	2			3,102,320
D	D	E	23111	2	2		625,874
D	D	E	23111	2			1,523,923
D	D	E	32111	2	2		1,169,939
D	D	E	32111	2			2,612,161
D	D	E	41111	2			598,694
D	D	E	122111	2	2		408,075
D	D	E	122111	2			5,577,676
D	D	E	131111	2			2,312,554
D	D	E	221111	2	2		272,004
D	D	E	221111	2			5,985,326
D	D	E	311111	2			1,767,903
D	D	E	1211111	2			2,720,156
D	D	E	2111111	2			1,087,704

D	E	E	12221		2		489,694
D	E	E	13211		2		761,878
D	E	E	22211		2		1,224,125
D	E	E	23111		2		897,808
D	E	E	32111		2		1,061,292
D	E	E	122111		2		1,224,015
D	E	E	221111		2		2,175,153
E	E	E	12221				680,228
E	E	E	13211				979,640
E	E	E	14111				108,864
E	E	E	22211				1,822,842
E	E	E	23111				326,462
E	E	E	32111				326,532
E	E	E	41111				27,216
E	E	E	122111				7,071,494
E	E	E	131111				1,904,191
E	E	E	221111				5,847,072
E	E	E	311111				816,240
E	E	E	1211111				8,700,292
E	E	E	2111111				2,718,452
E	E	E	11111111				1,630,704

5.4.6 Bands for patterns representing each number in the restricted eight-digit numbering space once only and separating patterns with attractive features and different quantities of distinct digits to avoid overlaps

Table 128 Bands for eight-digit patterns with repetitions, clusters and sequences separated out

Band assessed by…			Pattern				Capacity
DRCS	DRS	DS	Quantity of distinct digits	Length of the longest repetition	Length of the longest cluster	Length of the longest sequence	
A	A	A	1	8			9
A	A	A	8			8	6
A	A	B	2	7			162
A	A	B	2	6	4		81
A	A	B	2	6			324
A	A	B	2	5	4		162
A	A	B	2	5	3		162
A	A	B	2	5	2		324
A	A	B	2	5			324
A	A	C	3	6			1,944
A	A	C	3	5	3		1,296
A	A	C	3	5			10,368
A	A	D	4	5			18,144

B	B	B	2	4	4		324
B	B	B	2	4	3		972
B	B	B	2	4	2		648
B	B	B	2	4			243
B	B	B	2	3	4		1,053
B	B	B	2	3	3		1,944
B	B	B	2	3	2		810
B	B	B	2	2	4		1,053
B	B	B	2	2	3		1,377
B	B	B	2	2	2		243
B	B	B	2		4		81
B	B	B	7	2		7	15
B	B	B	7			7	90
B	B	B	8			7	33
B	B	C	3	4	4		1,296
B	B	C	3	4	3		9,072
B	B	C	3	4	2		12,960
B	B	C	3	4			28,512
B	B	D	4	4	3		13,608
B	B	D	4	4			176,904
B	B	D	5	4		5	23
B	B	E	5	4			136,057
B	C	C	3	3	4		5,184
B	C	C	3	2	4		34,344
B	C	C	3		4		1,296
B	D	D	4	2	4		27,216
B	D	D	4		4		81,648
C	C	C	3	3	3		46,656
C	C	C	3	3	2		90,720
C	C	C	3	3			43,416
C	C	C	3	2	3		136,728
C	C	C	3	2	2		152,280
C	C	C	3	2			10,368
C	C	C	3		3		31,104
C	C	C	3		2		8,424
C	C	C	6	3		6	19
C	C	C	6	2	2	6	19
C	C	C	6	2		6	274
C	C	C	6		2	6	188
C	C	C	6			6	532
C	C	C	7	2		6	190
C	C	C	7			6	1,044
C	C	C	8			6	240
C	C	D	4	3	3		45,360
C	C	D	4	3	2		381,024
C	C	D	4	3			879,984

C	C	D	5	3	2	5	23
C	C	D	5	3		5	274
C	C	D	6	3		5	297
C	C	E	5	3	2		108,841
C	C	E	5	3			2,068,142
C	C	E	6	3			816,164
C	D	D	4	2	3		580,608
C	D	D	4		3		430,920
C	D	D	5	2	3	5	48
C	D	D	5		3	5	425
C	D	E	5	2	3		163,248
C	E	E	5		3		816,055
D	D	D	4	2	2		2,648,268
D	D	D	4	2			1,579,284
D	D	D	4		2		725,004
D	D	D	4				127,764
D	D	D	5	2	2	5	909
D	D	D	5	2		5	1,534
D	D	D	5		2	5	1,411
D	D	D	5			5	1,128
D	D	D	6	2	2	5	302
D	D	D	6	2		5	5,467
D	D	D	6		2	5	2,644
D	D	D	6			5	9,671
D	D	D	7	2		5	2,175
D	D	D	7			5	10,842
D	D	D	8			5	1,977
D	D	E	5	2	2		4,489,731
D	D	E	5	2			12,082,370
D	D	E	6	2	2		680,079
D	D	E	6	2			15,643,459
D	D	E	7	2			3,807,860
D	E	E	5		2		4,434,797
D	E	E	6		2		3,399,168
E	E	E	5				4,271,784
E	E	E	6				15,638,997
E	E	E	7				11,418,744
E	E	E	8				1,630,704

Table 129 Prices in AED and categories for sample special mobile numbers in the benchmark countries

UAE Autotrader UAE	Australia Vodafone	Australia Telstra	Australia My-Number	Bahrain Batelco	Norway Gull-nummer	Oman Ooredoo	Pakistan Warid	Pakistan Nadeem	Qatar Ooredoo	Qatar Vodafone	Saudi Arabia Saudi Telecom	Singapore StarHub	Singapore M1	UK VIP Numbers
a**aaaa** 31,000	*aaaaa*** 789 Gold		*aaaaa*** 861 Gold		***aaaaa 4,795 Class C	*aaaaa** 954 Diamond	*****aaaaa 430 Platinum						*a**aaaa 2,160	***a*aaaaa 1,134
***aaaa** 15,000	***aaaa** 789 Gold		*aaaa**** 359 Silver	***aaaa* 1,948		**aaaa** 477 Silver	***aaaa*** 86 Gold		*aaa**a* 1,010		*aaaa**** 588	****aaaa 2,398 Tier 1		
	******aaa 144 Silver					*****aaa 477 Silver				aaa***** 1515 Diamond		*****aaa 1,318 Tier 2	****aa*a 405	
	***ababab 789 Gold	***ababab 1,148 Gold				**ababab 716 Gold								****ababab 1,134
		***aaabbb 1,148 Gold	aaa**bbb 1,435		**aaabbb 1,675 Class D	*aaa*bbb 716 Gold								aaa****bbb 113
*a*a**bbb 222	**aa*bbb* 144 Silver			**aa*bbb 974			*a*a***bbb 22 Silver	aa****bbb* 36	aa*bbb** 303				*a*a*bbb 543	
*****abab 350											****abab 284 Economy		****abab 408	
aa*baccab 100			*aaabcbc* 861 Gold	a*abcbbc 974				aa*abc*cbc 29		*abcacbc 1,010 Platinum				*aabcabca* 992
	***abcabc 789 Gold	***abcabc 1,148 Gold				**abcabc 477 Silver								****abcabc 851

6 Valuing numbers

This section uses the results from the benchmark countries provided in Section 3, the theory established in Section 4 and the techniques introduced in Section 5 to investigate ways of pricing numbers and the applicability of these ways in the UAE (particularly for mobile numbers). It shows how the patterns representing special numbers that could be appropriate to the UAE can be divided among potential price bands. It can be understood without understanding much of Section 4.

6.1 Price determination

6.1.1 Benchmarks for price matching

The following ways of valuing numbers seem to deserve consideration:

- Comparing prices of similar numbers in different benchmark countries (by building directly on the descriptions of the numbering practices in the benchmark countries).
- Devising algorithms to rank and price numbers according to their attractiveness (by adapting the classifications of numbers summarised in Section 5.2.4 and Section 6.2.4).

Table 129 above displays the prices (and the categories, if these are defined) of particular patterns of special mobile numbers that are offered by service providers and number traders in the benchmark countries. The following points are noteworthy:

- Numbers are offered for sale in almost all of the countries, but only in eleven of the countries are there sellers making available the prices of significant quantities of numbers. Fifteen such sellers are considered in the table.
- Many patterns of numbers are not on sale widely, even though they might be expected to be. As identical patterns are not on sale widely enough, the table makes comparisons between patterns that resemble each other loosely but are not identical. Even with such loose resemblances, on average fewer than 40% of the sellers offer patterns that resemble each other. The other sellers might not recognise the patterns as special enough to offer or might have no stock of suitable numbers to offer.
- The prices given for these patterns vary widely between countries and even within countries. The maximum price given for a pattern across all of the countries is on average 23 times the minimum price; for one pattern (requiring four, usually adjacent, occurrences of one digit) it is 174

times. Even within countries the maximum price given for a pattern can be twice the minimum price.

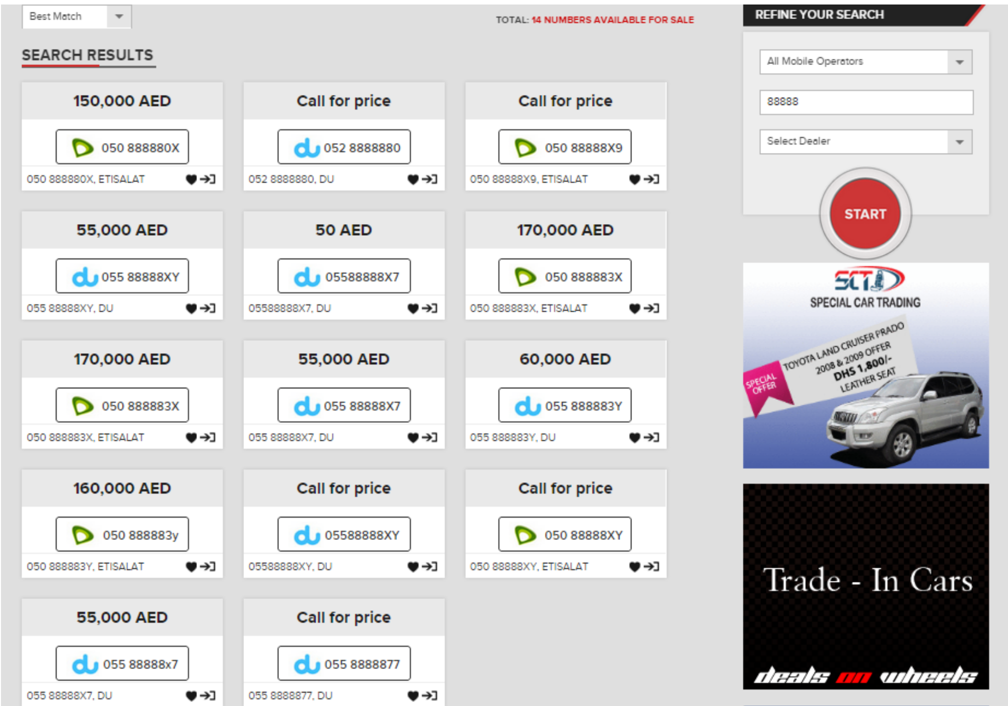
- The categories given for these patterns vary widely between countries and even within service providers. Thus a number containing '22222' is in the highest category in Oman but in the third highest in Norway; likewise a number containing '222' is in the highest category in Qatar but in the second highest in Australia, though the service provider is Vodafone in both cases. In the latter case the variation might be attributed to the difference in length (and therefore in rarity) of the numbers or to the difference in position of the repeated digits, but this explanation is not generally applicable.

- Some sorts of pattern appear to be attractive in some countries (or to some sellers) but not to others; among them are those containing '0' and those comprising many variants of ascending and descending sequences.

- As noted above, sellers have sometimes attached different prices to numbers that are different but that can be represented by the same pattern. A striking example is provided by Autotrader UAE, which appears to be selling '5588888*7' for both AED 50 and AED 55,000, as well as '55888883*' for AED 60,000 and '50888883*' for AED 160,000.

Figure 2 provides a screen shot for the Autotrader UAE prices illustrating this last point. Even if the price of AED 50 is dismissed as a misprint, the range of prices for apparently similar numbers is enormous; it is considered further in Section 6.3.2.

Figure 2 Special mobile numbers as priced by one number trader in the United Arab Emirates



In conclusion, the results of this study indicate that examining prices of comparable numbers in different benchmark countries is not likely to be useful for valuing numbers in the UAE. Comparable numbers are on sale in too few countries, and the prices of those that are on sale vary widely.

6.1.2 Algorithms for price calculation

The definitions and examples of capacities show that patterns that have approximately the same capacities might also have approximately the same attractiveness. The capacity of a pattern is potentially useful in measuring the attractiveness, because the capacity is directly related to the proportion of the numbering space that the pattern represents and therefore inversely proportional to the rarity of the pattern (if the numbers all have the same length).

Weakening the constraints on a pattern (by removing relations between the letters) increases the capacity and decreases the rarity. In fact:

- Putting '*' or an otherwise unused letter into a pattern while leaving the length of numbers unchanged increases the capacity (and decreases the rarity) by a factor that could be as high as ten.
- Increasing the length of all numbers just by adding '*' or an otherwise unused letter at the beginning increases the capacity of a pattern but leaves its rarity unchanged (relative to the size of the numbering space).

However, again the results from the benchmark countries provide no usable quantifiable evidence that can, for example, relate the price to the rarity by a power law of the form

price=constant×rarity^inflator.

Moreover, the definitions and examples of capacities suggest that:

- Patterns where repeated or sequenced letters are not adjacent might have the same capacities as patterns where repeated or sequenced letters are adjacent, but they do not necessarily have the same attractiveness.
- Patterns that include sequences might have the same capacities as patterns that include repetitions with the same length, but they do not necessarily have the same attractiveness.

Indeed, the attractive features of numbers do not appear to be directly tied to capacity. To determine these features, the numbering space is split following the procedures in Section 4.2.2 and Section 5.2.2. Doing this provides patterns that cover the numbering space fully but that have generally smaller capacities if they have more attractive features. There are too many such patterns for them to be treated separately when numbers are to be priced; they need to be banded together.

If the patterns in a band have approximately the same attractiveness as each other, the bands might be ranked broadly according to the value of numbers to customers. Even then, in the absence of market mechanisms there would still be no direct way of determining prices that capture the value of the numbers to consumers; however, the bands might be useful in setting relative reserve prices in auctions or for rebalancing numbering fees to provide the same total revenues while reflecting the ranking of the bands.

Ways of banding patterns together are investigated in Section 4.3.1, where bands are determined by pattern capacities, and in Section 5.3.1, where bands are determined by pattern features. The investigations show that determining bands by pattern features is the more promising of the two ways, so it is used below.

In conclusion, the results of this study indicate that relations between the prices of numbers and the attractiveness of numbers are no more precisely defined than the attractiveness itself. However, approximate measures of attractiveness might be used to place numbers in bands ranked broadly according to the value of numbers to customers. The bands could be determined by pattern features such as repetitions, clusters and sequences but not by pattern capacities.

6.2 Price setting

6.2.1 Possible annual fees

The intention here is to suggest annual fees for special numbers, if TRA chooses to set prices for numbers in bands so as to achieve the same total revenues as at present. Annual fees are preferable to one-off fees because:

- They minimise any disruption to the current practices of TRA and the service providers.
- They are fair to all persons: those who have already been assigned rights to use numbers are treated just like those who have yet to be assigned rights.
- They fit the requirements, discussed in Section 7.2.2, for any extra rights associated with special mobile numbers to lapse unless they are renewed.

Of course, annual fees could be converted into one-off fees by a discounted cash flow calculation that assumed an average length of time in which the rights to use special mobile numbers remained assigned. .

There is currently an annual fee of AED 2.0 for each allocated mobile number except ones with '0' in the second position, which are free. At most 40 million mobile numbers are allocated, so the revenues from mobile number fees amount to AED 72 million. According to *TRA Annual Report 2012*, total revenues from numbering fees are under AED 100 million.

Fees can be associated with numbers in each band if there is a required "base price" for one band and an expected relationship between the prices for different bands; for instance, fees might be related linearly or exponentially. Table 130 exhibits the fees that could be set for numbers in bands formed using each of the DRCS, DRS and DS rules discussed in Section 5.3.1. The relationship between the fees in different bands is exponential except in the case of band D and band E (which might be combined). For those two bands the fees are lower than the current flat rate fee, unless the bands are using the DS rule. For that rule the fees for band A are irrelevant, because the estimate below of the quantity of numbers allocated is zero. The DRCS band of a number is at least as high as the DRS band and the DRS band is at least as high as the DS band, so the fees become lower as the banding proceeds to the DRCS rule through the DS and DRS rules.

The exponential relationship between the fees in different bands could perhaps be related to pricing schemes for luxury goods. It matches very approximately the relationship between the rarity of numbers in the highest three bands (where the rarity of the numbers in a band is the inverse of the quantity of numbers in the band). It thereby ensures that roughly equal proportions of the revenues are generated by these three bands, on the assumption that equal proportions of the numbers in each band are allocated.

Table 130 Annual fees per number for bands

Band	Annual fee per number (AED) for band formed by...		
	DRCS	DRS	DS
A	1,000.00	1,200.00	
B	100.00	120.00	240.00
C	10.00	12.00	24.00
D	1.00	1.20	2.40
E	1.00	1.20	2.40

In conclusion, the results of this study indicate that the current annual fees for numbers could be rebalanced to charge more for numbers in higher bands and less for numbers in lower bands. Changing the annual fees in this way would be preferable to introducing one-off fees for special numbers, because it minimises disruption, is fair to everyone who has been or will be assigned numbers, and matches the requirement for rights of use to lapse unless they are renewed.

6.2.2 Possible annual revenues

Currently the blocks that are allocated have as many numbers as each other in each of the bands. This will continue to be so unless TRA starts to allocate numbers in punctured blocks (from which special numbers have been extracted for separate allocation, as discussed in Section 7.3.2). Indeed, it could continue to be so even if punctured blocks are introduced. It is assumed to be so here. Table 132 indicates the quantities of numbers (out of 40 million) in each band allocated and subject to fees, rounded down to the nearest 1,000, given that 10% of the numbers in the complete eight-digit numbering space are free of charge. The estimate is based on the capacities calculated in Section 5.3.2, which themselves sum those calculated in Section 5.4.3.

Table 131 Quantities of numbers allocated and subject to annual fees for bands in the complete eight-digit numbering space

Band	Quantity of numbers allocated and subject to annual fees in band formed by...		
	DRCS	DRS	DS
A	13,000	13,000	
B	216,000	154,000	4,000
C	2,725,000	1,945,000	251,000
D	19,860,000	17,242,000	3,101,000
E	13,184,000	16,644,000	32,643,000
Total	35,998,000	35,998,000	35,999,000

Table 133 shows the revenues that would result if these estimates for annual fees per allocated number (and for allocated numbers in the complete numbering space) were applied. The fees are designed so that roughly equal proportions of the revenues are generated by three of the bands, on the assumption that equal proportions of the numbers in each band are allocated.

Table 132 Annual revenues from numbers for bands in the complete eight-digit numbering space

Band	Annual revenue from numbers (AED) for band formed by…		
	DRCS	DRS	DS
A	13,000,000	15,600,000	
B	21,600,000	18,480,000	960,000
C	27,250,000	23,340,000	6,024,000
D	19,860,000	20,690,400	7,442,400
E	13,184,000	19,972,800	78,343,200
Total	94,894,000	98,083,200	92,769,600

The estimation above, for the complete numbering space, can be repeated for the restricted numbering space (except that 10% of the numbers are no longer free of charge). In this case it uses the classes of numbers listed in Section 5.4.6 instead of those listed in Section 5.4.3. The resulting estimates of the quantities of numbers allocated and subject to fees are rather similar to the corresponding estimates for the complete numbering space. Table 134 displays them.

Table 133 Quantities of numbers allocated and subject to annual fees for bands in the restricted eight-digit numbering space

Band	Quantity of numbers allocated and subject to annual fees for band formed by…		
	DRCS	DRS	DS
A	13,000	13,000	
B	214,000	154,000	4,000
C	2,725,000	1,945,000	251,000
D	19,862,000	17,242,000	3,101,000
E	13,184,000	16,644,000	32,642,000
Total	35,998,000	35,998,000	35,998,000

Table 135 shows the revenues that would result by using the estimates of numbers in each band for the restricted numbering space. They are very similar to those that would result by using the estimates of numbers in each band for the complete numbering space.

Table 134 Annual revenues from numbers for bands in the restricted eight-digit numbering space

Band	Annual revenue from numbers (AED) for band formed by...		
	DRCS	DRS	DS
A	13,000,000	15,600,000	
B	21,400,000	18,480,000	960,000
C	27,250,000	23,340,000	6,024,000
D	19,862,000	20,690,400	7,442,400
E	13,184,000	19,972,800	78,340,800
Total	94,696,000	98,083,200	92,767,200

In conclusion, the results of this study indicate that the bands determined by pattern features, and the prices appropriate to rebalancing the numbering fees, are robust enough to provide estimates of capacities and revenues that are broadly independent of the choice between the complete eight-digit numbering space and the restricted eight-digit numbering space in which '0' is never the second digit of a number.

6.3 Comparisons with other information

6.3.1 Regulatory fees abroad

The bands defined in Section 5.3.2 and the prices suggested in Section 6.2.1 can now be compared with those adopted in the UAE and elsewhere by regulators, service providers and number traders. As indicated in Section 6.1.1, there is very little consistency in the available data. The best that can be done is to select particular points of comparison. Those selected here come from the regulators in the neighbouring countries of Oman, Qatar and Saudi Arabia.

The regulator in Oman currently identifies three categories of special numbers (called 'diamond', 'gold' and 'silver'). Some choices of categories for the numbers are open to question, especially for the diamond and gold numbers; for instance, it seems that '27272749' is a diamond number but '22772277' is a gold number, '42272222' is a diamond number but '42224222' is a gold number, and '79123456' is a gold number but '87654321' is a silver number. The match with the bands defined in Section 5.3.1 is imperfect, because the special numbers identified by the regulator in Oman can be ranked highly if they have two shorter sets of adjacent occurrences of a digit instead of one longer set; examples of such numbers are the diamond number '22477779', the gold number '22247779' and the silver number '57224229'. In general, with the bands formed using the DRCS rule:

- The diamond numbers fit in band A, band B or band C: they have three distinct digits only, five-digit repetitions or four-digit repetitions and three-digit clusters.
- The gold numbers fit in band B, band C or band D: they have three or four distinct digits only, four-digit repetitions, three-digit clusters, four-digit clusters or ascending six-digit sequences.
- The silver numbers fit in band B, band C or band D: they have five distinct digits only, four-digit repetitions, three-digit repetitions, three-digit clusters, two-digit clusters or descending six-digit sequences.

For diamond numbers, gold numbers and silver numbers there are allocation fees of about AED 954, AED 71.55 and AED 28.35 respectively, and for normal mobile numbers there are allocation fees of about AED 0.24. These are allocation fees, not annual fees, so they are not strictly comparable with those above, but with a contract life time of two years (as provided by Etisalat for special numbers in the UAE) they are lower for some highly-ranked numbers than those in Section 6.2.1.

The regulator in Qatar does not identify special numbers and does not impose fees for numbers. It held a consultation in which it proposed to introduce fees, but it ultimately decided not to do so, as there would have been no clear benefit to the service providers and consumers.

The regulator in Saudi Arabia does not identify special numbers (except, now, for short codes) but did so at one period. The numbers were not banded or priced. Almost all of them require repetitions having at least four digits and therefore fit in band A or band B (with the bands formed using the DRCS and DRS rules) or sequences having at least five digits and therefore fit in band A, band B, band C or band D (with the bands formed using the DRCS, DRS and DS rules). There are also, for example, some numbers formed from particular clusters having at least three digits and therefore fitting in band B or band C (with the bands formed using the DRCS rule).

In conclusion, the results of this study indicate that the bands formed for the UAE are broadly applicable elsewhere but might match better those used in specific countries if the assumptions about favoured digits, repetitions, clusters and sequences were changed without changing the rules used for forming the bands.

6.3.2 Prices in the national market

The prices defined above can also be compared with those adopted by a number trader in the UAE, Autotrader UAE. A comparison with the Autotrader UAE data entails first removing the misprints and duplicates from the Autotrader UAE web site; at the time of writing 1,784 numbers were thereby reduced to 1,540. Table 136 presents the distribution of the numbers, banded according to the

DRCS, DRS and DS rules (horizontally) and according to price range (vertically). The table shows clearly that:

- The Autotrader UAE prices are spread very widely within each of the DRCS, DRS and DS bands. The spread appears to be inherent in the data: no rational banding, such as the one described in Section 4.3.1, seems to achieve a less dispersed result.
- There are several outliers, such as those at low prices for band A (which has a small sample). Even if the obvious outliers are removed the spread of prices remains very wide.
- Visually, the distribution of numbers for each band moves up the page (that is, towards lower prices) as its banding moves from A to E. To that extent the distribution broadly matches the bands formed using the DRCS, DRS and DS rules (or indeed those formed using the DRC and DR rules instead of the DRS and DS rules).

Table 135 Banding derived for numbers available from one number trader in the UAE

Price per number (AED)	Band formed by...														
	DRCS					DRS					DS				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
10-19					1					1					1
20-49															
50-99	1					1								1	
100-199	4		3	17	6	4		1	14	11			2	5	23
200-499			120	131	9			99	91	70			1	41	218
500-999			219	89	14			209	61	52			1	29	292
1,000-1,999			121	119	13			96	121	36			4	98	151
2,000-4,999			131	135	2			109	119	40			10	104	154
5,000-9,999		10	43	23	2		10	37	23	8			12	43	23
10,000-19,999		24	56	14			24	53	16	1		8	29	38	19
20,000-49,999	4	69	44	8	1	4	69	44	8	1		2	48	64	12
50,000-99,999	17	26	27			17	26	27				7	37	26	
100,000-199,999	12	3	2	1		12	3	2	1				11	7	
200,000-499,999	7	6				7	6						11	2	
500,000-999,999	3		1			3		1					3	1	
1,000,000-1,999,999	2					2					1		1		
Total	50	138	767	537	48	50	138	678	454	220	1	17	170	459	893
Proportion (%)	3	9	50	35	3	3	9	44	29	14	0	1	11	30	58

Table 137 allows comparison of the annual fees suggested in Section 6.2.1 with the Autotrader UAE prices for one-off sales. These are represented by the minimum, maximum, mean, median and first decile values for the bands formed using the DRCS, DRS and DS rules. It indicates that:

- The bands divide the Autotrader UAE numbers into sets of increasing size and decreasing average (and first decile) price.

- The annual fees of Section 6.2.1 are very well below the first decile values, and below the minimum values, of the one-off sales prices except in the case of band A, where there are outliers. In fact the first decile values of the one-off sales prices are more than thirty-one times the annual fees and the minimum values other than those outliers are more than twelve times the annual fees.

Table 136 Prices given for numbers available from one number trader in the UAE

Price per number (thousand AED)	Band formed by...														
	DRCS					DRS					DS				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
Maximum	1,400	300	700	130	20	1,400	300	700	130	20	1,000	99	1,400	700	45
Mean	186	47	8	3	1	186	47	8	3	1	1,000	39	73	16	2
Median	95	34	1	1	1	95	34	1	2	1	1,000	22	35	3	1
First decile	23	13	0.375	0.350	0.184	23	13	0.375	0.350	0.350	1,000	14	4.450	0.490	0.350
Minimum	0.050	5.500	0.100	0.100	0.011	0.050	5.500	0.150	0.100	0.011	1,000	13	0.160	0.050	0.011
Annual fee	1.000	0.100	0.010	0.001	0.001	1.200	0.120	0.012	0.001	0.001		0.240	0.024	0.002	0.002

In conclusion, the results of this study indicate that any proposed relationship between the fees for numbers in different bands should be considered in the light of views of customer demand as expressed by number traders and in consumer surveys and experiments. Moreover, discussions with the number traders would elucidate the effects on consumer behaviour and trading operations of any proposals to change number management for special numbers.

6.3.3 Special numbers abroad

Throughout much of this report eight-digit numbers receive most attention, as these are the mobile numbers in the UAE (following the initial '05') that customers might be able to choose. Their attractive features are taken to be:

- Few distinct digits, possibly separated by other digits (as in '24752725', '24742724' and '244224424', where there are respectively two, three or four distinct digits).

- Repetitions of two, three, four, five, six, seven or eight digits (as in '24222427', '22224427' and '22222447', where there are respectively three, four or five adjacent digits that are identical with each other).

- Clusters of two, three or four digits (as in '27724459', '27452479' and '27452475', where there are respectively two, three or four adjacent digits that are arranged in one cluster as an arbitrary permutation of the other cluster and there are at least two distinct digits in the cluster).

- Sequences that are either arithmetic progressions ascending or descending in steps of one (as assumed in Section 5.2.1) or sets of consecutive digits that may be repeated or permuted but must be selected so that sequences can be demarcated clearly (as assumed in Section 4.2.3).

These choices of attractive features are intended to be related to popular mobile numbers in the UAE, but they must remain slightly arbitrary. They make various assumptions; for instance, '786' is valued mainly in Bangladesh and Pakistan, so in the UAE the people who value '786' are probably low-income temporary workers and occurrences of '786' outside clusters are unlikely to increase greatly the value of numbers to potential customers. More generally, little is known about the willingness of customers to pay for special numbers in the UAE. Consumer surveys and experiments might invalidate the choices made above, which could well exclude patterns that some people like or include patterns that some people do not like. For instance:

- Particular favoured digits might be identifiable. In the Autotrader UAE list of numbers for sale, '0' occurs disproportionately often, by contributing 19% of the digits in the sixth and eighth positions (of eight-digit numbers) but not in most other positions; thus numbers ending with '2040' ("twenty forty"), for example, can have special pronunciations. In addition, numbers having '0' or '5' in the first position predominate (with respectively 74% and 17% of the total), because they are most of those that have been assigned (and perhaps because they match the '05' that precedes the eight digits). The Autotrader UAE list forms a small sample that is affected by the number reservation and assignment practices of the service providers; still, it suggests that customers favour particular digits in particular positions. If particular digits are favoured, numbers containing them might need to be placed in higher bands than other numbers, just as numbers that include six-digit sequences are placed in higher bands than almost all other numbers that have at least six distinct digits.

- Repetitions are assumed here to comprise adjacent occurrences of one digit each. However, in some countries numbers are ranked highly because they have repetitions of adjacent occurrences for two digits, not one; for instance, the regulator in Oman ranks '22477779' and '22497777' more highly than numbers with four-digit repetitions and four other distinct digits, such as '24777759'. An assumption suited to such cases, but offering a further generalisation, is that a repetition might comprise more than one part of a pattern with adjacent occurrences of a digit; the length of the repetition would be the sum of the quantities of digits in those parts (so for '22477779' and '22497777' the length would be six, not four, for '22477559' and '22497755' the length would be six, not two, and for '22447755' the length would be eight, not two).

- Clusters are assumed here to be arbitrary permutations, in the same number, of each other and to have at least two distinct digits each. However, the clusters in a number might be required to be identical (in which case '27452749' would be regarded as having a three-digit cluster even though '27454729' was not so regarded) or perhaps reversed (in which case '27454729' would be regarded as having a three-digit cluster even though '27452479' was not so regarded). Experiments might establish that the similarity between two clusters is difficult for people to recognise if the digits are permuted or reversed in one of them. Consequently the assumption that the clusters in a number must be identical might be preferable to the assumption that they could be arbitrary permutations of each other. Also, if a repetition could comprise more then one part of a pattern with adjacent occurrences of a digit, then a cluster should be able to comprise more than one part of a pattern arranged similarly elsewhere in the number; the length of the cluster would be the sum of the quantities of digits in those parts (so for '24247575' the length would be four, not two, as both '24' and '75' would contribute once). For the same reason the length of the cluster would be multiplied to reflect the quantity of similar arrangements of each part (so for '24247524' the length would be four, not two, as '24' would contribute twice). Indeed, the arrangements might be required to be adjacent to each other (in which case clusters would become generalisations of repetitions), or to be positioned near the beginnings or ends of numbers.

- Sequences are taken to be arithmetic progressions ascending or descending in steps of one (as described in Section 5.2.1) in several countries. However, there are exceptions; for instance, the regulator in Australia accepts steps of two as well as steps of one and the number traders in Norway and Pakistan single out numbers that include sequences in which digits are repeated or permuted (as described in Section 4.2.1).

In conclusion, the results of this study indicate that there could be assumptions about favoured digits, repetitions, clusters and sequences that are more appropriate to the UAE than those adopted in much of this report. If they were adopted, the numbers would need to be classified again, at least to the extent needed to put numbers into bands designed according to these more appropriate assumptions. However, the method of classification and the essentials of the final simple rules for forming bands would not be changed.

Table 137 Regulatory treatments of mobile numbers in the benchmark countries

Characteristic	UAE	Australia	Bahrain	Belgium	Egypt	France	Hong Kong	Ireland	Kuwait	Lebanon	Norway	Oman	Pakistan	Qatar	Saudi Arabia	Singapore	UK
Mobile number portability required	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Mobile number transfers permitted between service providers	Not explicitly banned	Yes	Yes	Only by approval	Yes	Only by approval	Yes	No	Yes	Only in licence transfers	Only by approval	No	Yes	Only by approval	No	Yes	Yes
Mobile number transfers permitted between end users	Not explicitly banned	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Only to some persons	Yes	Yes	No	Yes	Yes
Special mobile numbers defined by the regulator	No	No	No	No	No	No	Yes	No	No	No	No	Yes	No	No	Yes	Yes	No
Special mobile numbers defined by service providers	Yes	Yes	Yes	No	Yes	No	No	No	No	No	Yes	No	Yes	Yes	Yes	Yes	Yes
Special mobile numbers defined by number traders	Yes	Yes	No	No	No	No	No	No	No	No	Yes	No	Yes	No	No	No	Yes
Special mobile numbers traded by end users	Many	Many	Some	Some	Some	Some	Many	Some	Some	Many	Some	Some	Some	Some	Some	Many	Many
Extra fees charged by the regulator for special mobile numbers	No	No	No	No	No	No	No	No	No	No	No	Along with sales to customers	No	No	No	As parts of block fees	No
Extra rights given to end users holding special mobile numbers	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No

7 Number management policies

This section provides suggestions about the policy implications for the UAE of special number management. It reaches conclusions about buying and selling numbers, protecting consumers, and safeguarding the number supply that draw on the findings in the other sections (particularly those about the benchmark countries in Section 3). However, it can be read without reading those sections.

7.1 Buying and selling numbers

7.1.1 Number trading

The statement *Trading (buying and selling) in telephone numbers* by TRA in 2014 points out that customers who buy the rights to use numbers associated with particular packages are not buying the numbers themselves, which return to TRA (if the numbers are withdrawn) or the service providers (if the packages expire). This is so in other countries, too, regardless of whether number trading is prohibited: formally, number trading relates to buying and selling the rights to use numbers, not the numbers themselves.

Table 138 above summarises some of the information about regulatory policies in the benchmark countries. It might give the impression that several countries prohibit or limit number transfer and therefore number trading. However, number transfer "only by approval" in the table relates to transfers other than assignments (or 'secondary allocations') from service providers to other service providers or end users. Some regulations (in Lebanon, for example) distinguish assignments from transfers carefully; others (in Bahrain, for example), regard assignment and porting as forms of transfer, and thereby legitimise transfer. Elsewhere often the notion of transfer is not defined, but it appears in its context to refer to moving the primary allocations of numbers from one service provider to another, not just to assigning rights of use; number trading, however, could happen when service providers assign numbers (or, formally, the rights to use numbers) for payment without moving the primary allocations.

The reason for a prohibition on number trading is unclear; it might be a belief that "national resources" should not be traded, or it might be administrative convenience. The prohibition is easy to circumvent, as discussed below. Moreover, land, which is the most obvious national resource, can be transferred or traded in most countries (if only on long leases). Also, domain names, which are analogous to phone numbers in several respects, can be transferred or traded in many countries.

In the UAE, the domain name policies *Domain Name Transfer – Change of Registrar* and *Domain Name Transfer – Change of Registrant* lay down fully the circumstances in which domain names can be transferred. However, transferring domain names is analogous more to porting phone numbers than to performing secondary allocation. The policies do not mention buying or selling domain names (except for specifying when fees may be charged for transfers) but the domain name *Code of Practice* proscribes hoarding them.

National Numbering Plan 2008 by TRA describes the connection between transferring numbers and transferring licences, but it does not prohibit number trading. Moreover, number trading occurs in all of the benchmark countries, even in those few where it is prohibited. Indeed, prohibition is almost irrelevant, as there are many ways the prohibition can be circumvented. For instance:

- The service providers can include special numbers as benefits in tariff packages and loyalty schemes.
- The service providers can pass to distributors sets of SIMs associated with blocks of numbers, under their usual terms and conditions.
- The distributors or further resellers can sell to their customers SIMs associated with special numbers.
- Customers can sell to other customers SIMs bought by them.

Other ways of circumventing the prohibition could easily be devised, even if SIMs were not sold; for instance, "shell" companies (which do not sell goods or services but which nonetheless participate in transactions) could be registered, assigned numbers by the regulator and sold if the numbers were valuable enough to justify the administrative and financial overheads. This has indeed happened in the United States (US), where number trading is formally prohibited. Because the prohibition is easy to circumvent, in this report a "number trader", such as a distributor or further reseller that sells SIMs associated with special numbers, is not necessarily doing something illegal if number trading is prohibited.

An explicit prohibition of number trading, even if it were enforceable, would not offer any obvious benefits. In fact a lively and disciplined secondary market would help with distributing numbers to the people who find them most attractive. It could also provide a useful stimulus to service providers, both by reducing the prices of special numbers and by reducing the take-up of loyalty schemes that preclude switching between service providers.

In conclusion, the evidence in this study suggests that number trading is not obviously detrimental (and could even be beneficial) to consumer well-being and should not be subject to regulatory measures, especially as international experience indicates that such measures are difficult to enforce.

7.1.2 Number fees

The investigations in this report confirm that in the benchmark countries there are similar opinions about which numbers are more or less attractive. However, they also show that:

- Comparing prices of similar numbers in different benchmark countries is not likely to be useful for setting prices for numbers in the UAE. There are very large variations in prices between countries, and sometimes even within countries, that prevent the emergence of standard prices, even though markets in numbers are well established.
- Devising algorithms to rank numbers according to their attractiveness can place numbers in bands that might have roughly similar prices.

Placing numbers in bands is not the same as determining the prices. There remains no convincing way of determining the prices of numbers administratively (without reference to markets), if those prices are to capture the value of the numbers to consumers. To determine such prices regulators need to know how much consumers would pay for the numbers. Regulators could instead use price bands as a way of varying the numbering fees intended to cover regulatory costs, so that lower fees were paid by customers who did not want special numbers. In other countries even this does not happen for mobile numbers: the activities of regulators related to special numbers are very limited and largely concerned with short codes and some other numbers having clear commercial significance. The following factors might be responsible for this:

- An administrative allocation process is likely to lead to inefficiencies in allocation, because of the difficulties in valuing numbers.
- Managing special numbers separately from other numbers creates unnecessary administrative burdens for regulators and service providers.
- Making distinctions between special numbers and other numbers could impede the main regulatory activity of managing the numbering space.
- Too much regulation could stop secondary markets in special numbers from distributing numbers efficiently and stimulating service providers.

Among the benchmark countries, the liveliest secondary markets appear to be those such as Australia, Norway and the UK where the regulator does not impose fees for special mobile numbers. There are other possible explanations of why some markets are livelier than others; nonetheless, overall, extra fees for special mobile numbers may not support the development of the market and are likely to be very difficult to determine administratively.

Federal Telecommunications Law 2003 as amended does not mention safeguarding national resources as an objective for TRA. However, the statement *Our Vision, Mission and Values* by TRA provides as a strategic goal "Striving to ensure the provision of the required resources in a fair manner to develop the best services in the ICT sector". Raising revenues and appropriating some of the value of national resources are not given as goals. If, however, they are goals, then there could be better approaches to reaching them than charging extra fees for special numbers. One is for TRA to take a proportion of the selling prices of all new numbers, leaving the service providers and number traders to decide whether to sell the numbers at fixed prices or in auctions (or indeed as components of tariff packages, which would need to be broken open to establish the prices). The sum would be paid by the service providers through their customer registration processes. However, this could seem like a tax on the mobile sector and so could affect how outside observers saw the UAE market. Indeed, effectively special numbers would be being taxed as luxury goods, though there is currently no sales tax in the UAE.

In conclusion, the evidence in this study suggests that the value of special numbers to consumers is difficult to determine without recourse to market mechanisms such as auctions and sales by number traders. Extra fees for special numbers could be created by rebalancing the existing fees to reflect the value broadly while leaving the total revenues to TRA and overall prices to consumers unchanged; revised revenue requirements could be met by scaling the rebalanced fees appropriately. Nonetheless, markets themselves could be more effective than administratively determined fees in ensuring recognition of the value in national resources, and even in raising revenue.

7.1.3 Number auctions

An alternative to the introduction of extra administratively determined fees for special numbers would be the introduction of auctions. Small quantities of numbers from the bands provided in this report would be auctioned at regular intervals or according to demand; reserve prices, or at least guide prices, would be guessed for the bands in the first auction and refined in later auctions. The whole process could take place online, as in Singapore.

Auctions would not necessarily require TRA to have extensive processes and systems for individual number allocation: instead, TRA would collect the proceeds of the auctions while the service providers would assign the numbers to the purchasers. However, auctions of individual numbers by TRA would entail entering the market for numbers while controlling the supply. Monopolies of supply by regulators are delicate matters: they have stimulated (for example) attempts to introduce US phone numbers outside the North American Numbering Plan (NANP) and top-level internet domains outside the control of the Internet Corporation for Assigned Names and Numbers (ICANN).

A further possibility for auctions is the auction of blocks, not individual numbers, by TRA. The participants in the auctions could include not just the two service providers but also any number traders that had the money and staff to compete against the two service providers. Nonetheless, such auctions would tend to favour the service providers over the number traders, because of their financial resources. The design of the auctions would need to prevent anti-competitive behaviour; for instance, as is said to have happened in the Netherlands, in a second-price auction for special numbers a large service provider might bid a much higher price than its small competitors could possibly afford, in order to squeeze them out but pay only the second, next highest, bid price.

Regulators rarely enter the market for numbers. In Singapore there can be intervals of several years between auctions; administrative allocations are usual. The regulators in other countries, such as Hong Kong, Oman and the US, have considered introducing regulatory auctions but have not yet chosen to do so. Moreover, the regulator in Australia, long noted for auctions of toll free and local rate numbers (not mobile numbers), has now replaced its auctions by administrative allocations.

Auctions by TRA would increase the administrative costs of TRA (and perhaps the service providers). They could well increase the prices ultimately paid by consumers for special numbers, because the auction participants would be competing against each other instead of buying in numbers at fixed prices.

The justification for the regulator benefitting financially from special numbers seems to be based on wanting to avoid "giving away" national resources. However, the current system seems to work. If it provides choice for consumers without adverse effects on consumer well-being, competition or the number supply it does not need to be changed.

In conclusion, the evidence in this study suggests that regulatory auctions for special numbers would not be obviously preferable to the current system in the UAE. Introducing them would require care to avoid giving the impression of competition between the regulator and the service providers and number traders, artificially limiting the supply, disrupting the current secondary market, and raising overall prices paid by consumers.

7.2 Protecting consumers

7.2.1 Rights to use general numbers

National Numbering Plan 2008 by TRA, *Consumer Protection Regulations 2014* by TRA and the web sites of the service provider in English do not describe the rights to use numbers that are assigned to customers. In particular, they do not describe how the rights change when customers do not maintain their accounts actively. The changes can be quite complicated. For instance, the holder of a time-limited prepaid account can find one of the following situations:

- Calls can be received from all numbers and can be made to all numbers, because the account contains credit and does not yet need renewal.

- Calls can be received but can be made only to numbers that are free to the caller, because the account does not contain credit but does not yet need renewal.
- Calls can be received from all numbers but can be made only to numbers that are free to the caller, because the account needs renewal; any credit in the account would become usable again after renewal.
- Calls cannot be received or made, because the time available for renewal has expired; any credit in the account has been lost, along with the number.

All customers should be informed in simple terms what the rules are for their accounts, regardless of whether they have bought their numbers. In particular, service providers and number traders should tell customers clearly and simply, in advertising and before sales, that numbers will be kept only if accounts are maintained (by topping up the credit or otherwise). Moreover, consumer protection might point to prohibiting some rules that could be unfair, unexpected or difficult to understand and remember.

In the UAE there appear to have been purchasers of special numbers who were unaware of how they can lose their numbers. The lack of awareness might be widespread. If so, TRA should use its powers of regulation and persuasion to ensure that service providers and number traders tell customers how they can lose their numbers. *Federal Telecommunications Law 2003 as amended* does not obviously give TRA powers to impose any necessary regulation on number traders. However, the Consumer Protection Department could be asked to do so. The Consumer Protection Department would then also point out that selling SIMs beyond their "activate-by" dates is, formally, like selling food beyond its "sell-before" date and that, as stated in *Trading (buying and selling) in telephone numbers*, purchasers own the rights to use numbers, not the numbers themselves.

In conclusion, the evidence in this study suggests that TRA and perhaps the Consumer Protection Department might need to codify and strengthen the rights of customers, particularly in relation to numbers.

7.2.2 Rights to use special numbers and other desirable numbers

Customers obtain no extra rights to numbers by buying special mobile numbers in any of the benchmark countries. In particular, they do not obtain lifelong rights of use: their rights continue to be those in the terms and conditions of whatever tariff packages they buy along with the numbers. Indeed, only in Australia do they obtain extra rights by buying special numbers of any type; the rights there apply only to certain toll free and local rate numbers and permit leaving numbers inactive for up to three years.

Extra rights for customers who buy special mobile numbers would need to let customers keep the numbers but would also need to avoid losing the numbers from the number supply forever. More generally, these rights would need various attributes that on closer inspection appear to be more of an encumbrance to consumers than a protection. In particular:

- Such rights should apply to numbers chosen for their personal significance as well as to other special numbers. Consumers get attached to, and are identified by, their numbers. Consequently any number could acquire personal significance and should be eligible to be retained indefinitely. The current way of ensuring this is to maintain the account. It is understood widely; introducing extra rights for special cases might just produce extra complications.
- Such rights should lapse after some time unless renewed (much as happens with domain names), to avoid leakage from the number supply and to permit re-assignment of special numbers that have been assigned but are not used. Customers who did not renew the rights would lose the numbers. However, this is exactly what happens currently, as numbers are already lost after some time if accounts are not maintained. Consequently the extra rights would offer no extra security to the careless and would just impose an extra burden on the careful.
- Such rights should be granted automatically through the SIM registration process for sales of new numbers. Even so, customers might be confused, because the times by which accounts must be renewed could differ from the times by which the extra rights must be renewed. Customers might also find that renewing the extra rights was complicated, if, for example, the numbers had been ported and the account had later not been renewed.
- Such rights should also be available to consumers that happen to have already obtained numbers that they are reluctant to lose. As already noted, consumers get attached to, and are identified by, their numbers; they also want to avoid having to tell all of their contacts that their numbers have been lost or changed. Consequently the extra rights should be available even if customers have not bought their numbers; there would need to be another process, beside the SIM registration process, for obtaining the rights. Again this would complicate matters further.

In conclusion, the evidence in this study suggests that granting extra rights to customers who buy special numbers could be confusing and complicated. It could also be ineffective: customers would need to renew their rights from time to time (as assigning numbers in perpetuity would cause leaks from the supply) and would forget to do that, just as they forget to renew their subscriptions to the services. Effort could be devoted instead to codifying and strengthening the rights available to all customers, even if they do not buy special numbers.

7.3 Safeguarding the number supply

7.3.1 Number hoarding

If number trading is permitted, there is perhaps a risk of number hoarding: number traders might obtain and retain "too many" numbers and might then:

- Impede the development of telecommunications services such as ones in which numbers are read or pronounced as words.
- Extort high prices from customers who just wanted to choose numbers that were special for personal reasons.
- Place pressure on the regulator to release more numbers for allocation even though the utilisation of the allocated numbers was extremely low.

Mobile numbers that have attractive patterns of digits are fairly rare, so number traders are unlikely to have much effect on the overall number supply in the UAE by amassing only such numbers. Any effect could be mitigated in the following ways:

- The service providers would require distributors and other resellers of numbers to pay the annual fees for numbers and to achieve high utilisation of existing stocks before passing on any more numbers. In the UAE the annual fees are high, and should act as a large disincentive to number hoarding.
- The process for recycling unused numbers would be documented, monitored and implemented by all parties. In particular, number traders that acquired large blocks of undifferentiated numbers in order to extract special numbers would pass them on for resale instead of discarding them. Similarly service providers to which numbers had been ported would ensure that numbers were returned to the range holders when customers had stopped using them.
- Number traders that held enough of the number supply to extort high prices would need to be subject to general competition law (in the absence of specific powers held by TRA).

In the US there are regulations to prevent the hoarding of toll free numbers. Nonetheless no action has been taken against a particular company that with its affiliates has amassed about 25% of the toll free numbers. Other companies have suggested that the resulting pressure on the number supply could be relieved by introducing auctions of toll free numbers like those that have now been abolished in Australia; however, the hoarding of toll free numbers is prevented in Australia not by holding auctions but by reclaiming numbers that are left inactive for more than three years.

In conclusion, the evidence in this study suggests that number hoarding is not likely to cause problems for the mobile number supply in the UAE and could be controlled by existing regulatory measures. Indeed, it has not been noted as occurring in the UAE.

7.3.2 Punctured blocks

Regulators typically allocate most numbers in blocks. They might remove some numbers from the blocks before allocation, in which case the blocks are punctured. Punctured blocks arise if regulators allocate numbers individually or prohibit assignments of certain numbers from allocated blocks (such as numbers that look very similar to emergency service numbers).

Discussions with the service providers in the UAE confirm that there appears to be no reason why the networks could not accommodate punctured blocks. There might possibly be reasons why business support systems and operations support systems could not do so without modifications; the service providers would need to check. However, the modifications to the systems caused by the introduction of number portability will probably have removed almost all potential problems.

Currently punctured blocks arise when TRA allocates numbers individually. It does this on a small scale now (for toll free numbers), but the operation would need to be greatly enlarged if TRA were to start assigning mobile numbers to individual customers. The service providers already have extensive processes and systems that do this and that provide necessary support services, such as customer care and number portability. There is no obvious merit in having TRA duplicate these processes and systems.

In conclusion, the evidence in this study suggests that allocating punctured blocks is technically feasible but is not especially desirable. Allocating whole blocks, and using the existing processes and systems of the service providers for assigning individual mobile numbers, can achieve the same result.

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